Supersonic collapse of Langmuir waves

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1. The collapse[1–5] of Langmuir waves play a fundamental role in the physics of plasma turbulence, and determines in many cases the mechanism whereby energy is transferred from Langmuir waves to the plasma particles. The most fundamental problem in the theory of collapse is that of the regime of strong compression of Langmuir caverns, where the oscillation energy density in the caverns ($w=E_0^2/4\pi$) ranges from $w_0=(m/M)nT$ to $w_{\max}\sim nT$, after which Landau damping or intersection of electron trajectories sets in.

The Langmuir collapse is described by the system of equations[1]

$$\Delta(2i\,\psi_t+\Delta\psi)=\operatorname{div} n\,\nabla\psi\,, \qquad (1)$$

$$n_{tt}-\Delta n=\Delta|\nabla\psi|^2. \qquad (2)$$

Here $n=(3/8)(\delta n/n_0)(M/m)$ is the dimensionless variation of the plasma density, $t=(4/3)(m/M)\omega_p t$ and $\mathbf{r}=(4/3)\sqrt{m/M}\,(\mathbf{r}/r_D)$ are dimensionless coordinates, and ψ is the envelope of the high-frequency potential

$$\nabla\phi=\frac{8}{3}\left(2\pi n_0 T_e\frac{m}{M}\right)^{1/2}\operatorname{Re}\nabla\psi e^{i\omega_p t}.$$

For motions with characteristic space and time scales L and τ, satisfying the conditions

$$\begin{aligned}&1\gg L\gg\tau\\&1\gg\tau\gg L^2\end{aligned} \qquad (3)$$

it is possible to go over in (1) to the adiabatic approximation

$$i\,\psi_t\to-\lambda_0^2(t)\,\psi\,,$$

and Eq. (2) can be simplified to

$$n_{tt}=\Delta|\nabla\psi|^2 \qquad (4)$$

After these simplifications, the equations admit of a self-similar formulation[1]

$$\lambda^2(t)=\frac{\lambda_0^2}{(t_0-t)^{4/3}};\quad \mathbf{r}=\vec{\xi}\,(t_0-t)^{2/3};\quad \vec{\psi}=\frac{1}{(t_0-t)^{1/3}}R(\vec{\xi}\,)$$
$$n=(t_0-t)^{-4/3}V(\vec{\xi}\,) \qquad (5)$$

corresponding to collapse at $t=t_0$. For this substitution we have $L\sim\tau^{2/3}$ and the conditions (3) improve as $\tau\to 0$. In this case $w/nT\sim(m/M)\tau^{-2}\gg m/M$, i.e., the intensity of the Langmuir waves is contained in the interval of interest to us.

In the regime (5), the characteristic dimension Δr of the cavern and the characteristic time Δt of its compression satisfy the condition $\Delta r/\Delta t\gg c_s$ (c_s is the ion-sound velocity), so that one can speak of supersonic Langmuir collapse. The equations for $R(\vec{\xi})$ and $V(\vec{\xi})$ have solutions if the potential R is antisymmetrical, $R(r,z)=-R(r,-z)$. In the regime (5) there is produced at $t=t_0$ a singularity of the type

$$|\nabla\psi|^2=\delta(r)$$

and the entire energy contained in the cavern is gathered into a point.

2. The systems of equations (1), (2), and (1)–(4) were solved by us numerically in axially symmetrical three-dimensional geometry. It was shown earlier[5] that a collapse takes place within the framework of the system (1) and (2). The description of the corresponding formulations of the problems can be found in[4,5]. New numerical calculations have shown that under the initial conditions $|\nabla\psi|^2\sim 10-20$ ($\omega/nT\sim(10-20)m/M$) the behaviors of the solutions of the systems (1), (2), and (1)–(4) do not differ qualitatively. In either case one observes the formation of a collapsing cavern of the dipole type (see[4,5]), the evolution of which in time satisfies the self-similar relations (5) with good accuracy (see Fig. 1). The deviation of the functions $|E|^{-1}(t)$ and $|n|^{-3/4}(t)$ from linearity near $t=t_0$ is due to the fact that the approximation of the differential equations (1) by difference schemes no longer holds near the collapse.

3. The collapse develops as a result of instability of the Langmuir condensate. If the condensate intensity is $w_0\ll(m/M)nT$, the collapse is subsonic during the first stage ($\Delta r/\Delta t\ll c_s$), and the supersonic regime sets in at $w\sim(m/M)nT$. The dimension of the cavern is then $\Delta r\sim\sqrt{M/m}\,r_D$, and the energy contained in the cavern is

$$\Delta\epsilon\sim nTr_D^3\sqrt{\frac{M}{m}}\,.$$

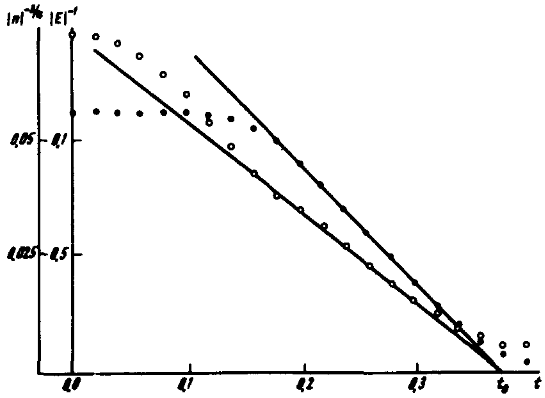

FIG. 1.

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The cavern shrinks in self-similar fashion until the oscillation energy in it becomes comparable with the thermal energy ($w \sim nT$), and then its dimension is

$$r_{min} \sim \left(\frac{M}{m}\right)^{1/6} r_D .$$

In a Maxwellian plasma, the Landau damping of waves with $k \sim 1/r_{min}$ is still exponentially small, and the cavern continues to be compressed to dimensions $r < r_{min}$. The oscillator velocities of the electrons in the cavern then exceed the thermal velocity, and this makes it necessary to take into account the electronic nonlinearities. The energy dissipation in such a cavern is due to the intersection of the electron trajectories. If the intensity of the Langmuir condensate w_0/nT then exceeds m/M, then the collapse is supersonic from the very beginning. The initial dimension of the cavern is now $r_0 \sim r_D (w_0/nT)^{-1/2}$, and $\Delta\epsilon \sim nTr_D^3(nT/w_0)^{1/2}$.

If $w_0/nT < [\ln(m/M)]^2$ (we have $w_0/nT < 10^{-2}$ for a deuterium plasma), the cavern will radiate energy as before, owing to the trajectory intersection, whereas in the opposite case the energy absorption is due to Landau damping when the cavern reaches the minimum dimension $r_{min} \sim r_D$.

In a plasma containing accelerated particles (tails of the distribution function), the role of the Landau damping in the absorption of the cavern energy becomes much more significant. We note that Langmuir collapse in itself is a strong mechanism for the formation of high-energy "tails."

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