Anisotropic thermal effect in superconductors

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An expression is obtained for the field produced in the anisotropic thermal effect (ATE) in superconductors. It becomes possible to observe the ATE in highly purified samples, in which the principal role is played by the relaxation mechanism; this agrees with recent measurements.

Selzer and Fairbank[1] recently reported that in pure superconducting Sn there is produced a magnetic field due to a temperature gradient. This raises the question of the nature of the observed effect. We consider here the anisotropic thermal effect (ATE) produced in superconductors by an electron-phonon relaxation mechanism. The calculations and the corresponding estimates show that Selzer and Fairbank were apparently the first to observe the ATE in[1].

The ATE was first considered back in 1944 by Ginzburg.[2] When a temperature gradient acts in a sample there is produced, in addition to the heat flux, also a normal current. However, in an isotropic superconductor this current is completely offset by the superconducting current. In an anisotropic crystal there is in general no such compensation; a circulating current is produced together with an associated magnetic field.

We have previously[3,4] considered the ATE on the basis of the microscopic theory of superconductivity. We investigated an electron-impurity scattering mechanism. It was shown that the effect increases as T_c is approached and with increasing mean free path λ. In this connection, measurements were made[1] on very pure tin ($\lambda \sim 0.1$ cm) at $T \sim T_c$ up to $T/T_c = 0.99$. At such large mean free paths, however, the principal role is assumed by the scattering of electrons by phonons (electron-impurity scattering, as is well known, makes a contribution at $\lambda \lesssim 10^{-2}$ cm; see, e.g., [5]). We consequently consider below the electron-phonon mechanism of the ATE.

Consider a uniaxial sample (Fig. 1) in which ∇T is produced. We are considering the homogeneous samples; inhomogeneous systems are treated in[6–8]. The self-consistent system of equations describing the normal and superconducting currents is given by

$$j_\alpha^n = b_{\alpha\beta} \nabla T; \quad j_\alpha^s = K_{\alpha\beta}(q) A_\beta(q) \qquad \operatorname{rot} \mathbf{H} = \frac{4\pi}{c}(\mathbf{j}^n + \mathbf{j}^s) \tag{1}$$

$K_{\alpha\beta}$ is the Pippard tensor and $b_{\alpha\beta}$ are the thermoelectric coefficients.

The solution of the system (1) leads to the following expression for the sought magnetic field:

$$H_z(y) = -\frac{4\pi}{c}\frac{d}{dy}[K^{-1}(0)\, j^n(y)] \tag{2}$$

The choice of the axis is indicated in Fig. 1; $j^n(q) = j_x(q) - (K_{xy}/K_{yy}) j_y^n(q)$.

Formula (2) can be reduced to the form

$$H_z(y) = -\frac{4\pi}{c}(\nabla T)^2 \sin 2\theta \frac{dL}{dt} \tag{3}$$

where θ is the angle between the crystal axis and the s axis (see Fig. 1), $L = (b_\parallel/K_\parallel) - (b_\perp/K_\perp)$; $b_\parallel$ and $b_\perp$ are the principal values of the tensor $b_{\alpha\beta}$ (for example, $b_\parallel$ corresponds to $\nabla T \parallel s$).

Further, as seen from (2) and (3), the problem reduces to a determination of the current, and consequently of the values of $b_\parallel$ and $b_\perp$. To this end it is necessary to find the increment to the distribution function of the electronic excitations. It is determined from the kinetic equation

$$-\frac{\epsilon}{T}\frac{\partial f_0}{\partial \epsilon}\frac{\partial \epsilon}{\partial \mathbf{p}} \nabla T = I_{\mathrm{col}} \tag{4}$$

$\epsilon = (\xi_{\mathbf{p}}^2 + \xi_{\mathbf{p}}^2)^{1/2}$ is the energy of the electronic excitations, and I_{col} is the excitation-phonon collision integral. The corresponding Hamiltonian is

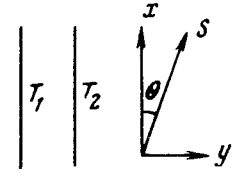

FIG. 1.

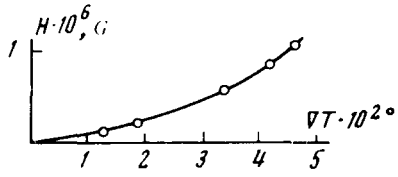

FIG. 2.

$$H' = \sum_{k,q} V_{kk'} \{ (u_k u_{k'} - v_k v_{k'})(a^+_{k0} a_{k'0} + a^+_{k1} a_{k'1}) + (u_k v_{k'} + u_{k'} v_k) \times (a_{k1} a_{k'0} + a^+_{k0} a^+_{k'1}) \} b^+_q + \text{c.c.}$$

The usual (u,v) transformation is employed. In the considered case of an anisotropic crystal, I_{col} can be written down in analogy with the procedure used in the isotropic case.[4]

The kinetic equation for the distribution functions of electrons interacting in a normal metal with phonons, in the presence of a temperature gradient, was investigated by Landau and Pomeranchuk[9] in connection with the analysis of thermoelectric phenomena. The method developed in[9] can be generalized to include also the investigated case of an anisotropic superconductor. We note that to estimate H we can use also a direct generalization of[9] to include the anisotropic case, since we are interested in temperatures that are quite close to T_c.

In view of the continuity of the kinetic coefficients, it is clear that the values of $b_{\parallel}^{sup}$ and $b_{\perp}^{sup}$ are close to the values of $b_{\parallel}^{norm}$ and $b_{\perp}^{norm}$.

The solution of (4) can be written in the form $f = f_0 + f_1$, where

$$f_1 = -\frac{\partial f_0}{\partial \epsilon} \frac{\beta_1(\Omega)}{T^4} \frac{v \nabla T}{v} \tag{5}$$

and β_1 is a function that depends only on the angles. The current j^n in the considered anisotropic superconductor is calculated from the formula[6]

$$j^n = -2e \int \frac{\partial \xi}{\partial \mathbf{p}} f_1 \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} \tag{6}$$

$\partial \xi / \partial \mathbf{p}$ is the electron velocity in the normal metal (in[3] we used the expression $\mathbf{j}^n = -2e \int \mathbf{v}^s f_1 d^3\mathbf{p}/(2\pi\hbar)^3$, $\mathbf{v}^s = \partial\epsilon/\partial p$, which led to an overestimate of the field in the case of electron-impurity scattering considered in[3]). The validity of (6) can be verified by performing a (u,v) transformation in the expression for the current operator.[6] Calculating further the sought field, with allowance for (5), (6), and (3), we arrive at the formula

$$H_z = -\frac{4\pi}{c} (\nabla T)^2 \frac{\sin 2\theta}{T^4} \left[\frac{C_{\parallel}}{K_{\parallel}} - \frac{C_{\perp}}{K_{\perp}} \right] T_k^{-1} \left(1 - \frac{T}{T_c} \right)^{-2} \tag{7}$$

where

$$C_{\parallel,\perp} = \frac{2e}{(2\pi\hbar)^3} \int \frac{d\sigma}{v_F^2} v^2_{\parallel,\perp} \beta_1(\Omega).$$

$b_{\parallel,\perp}^{norm} = C_{\parallel,\perp}/T^4$ are the principal values of the thermoelectric coefficients in the normal metal.

We now estimate with the aid of (7) the possible values of the field in the case of ATE

$$H_z = -\frac{4\pi}{c} (\nabla T)^2 \delta^2_{L0} b^n T_c^{-1} (1 - T/T_c)^{-2} \tag{8}$$

δ_{L0} is the London depth of penetration at $T = 0°$, $b^n = \alpha\sigma$ is the thermoelectric coefficient in the normal metal, α is the differential thermoelectric power, and σ is the conductivity.

Substituting in (8) the values $\sigma = 10^{22}$ cgs esu,[5] $\alpha = 10^9$ cgs esu/g,[8,10] $\delta_{L0} = 5 \times 10^{-6}$ cm, $T = 3.68°$, $T_c = 3.72°$, and $\nabla T \approx 4 \times 10^{-2}$ g/cm (these values characterize the state of the pure Sn samples used in[11]), we get $H \approx 10^{-6}$ G. It is this value of H which corresponds to the value of the field observed in[11] (see Fig. 2, which shows the data of[11]). The dependence of v on ∇T also agrees with the theory. Thus, Selzer and Fairbank were apparently the first to observe ATE.

The value of H for ATE in samples measuring ~1 cm corresponds to several magnetic-flux quanta. Therefore measurements of ATE in nonsingly-connected samples can lead to observation of flux quantization (although the values of H will be somewhat different because of the change in the geometry of the sample). H can be varied either by lowering T or by contaminating the sample. It is of interest therefore to perform the corresponding experiments.

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