Mechanism of excitation of combination frequencies in ionospheric plasma

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We consider the mechanism whereby combination frequencies are excited in an ionospheric plasma acted upon by powerful short-wave radiation. The results are compared with experiment.

The first results of experiments on nonlinear detection of modulated short-wave signals in the ionosphere were reported in[1], where the most probable mechanism was also suggested. This mechanism was based on thermal effects and on modulation of the ionospheric current systems under the influence of high-power radio emission. We present below a quantitative theory of this phenomenon, which confirms the qualitative conclusions of[1].

The formulation of the problem is the following: an electromagnetic wave of frequency ω and amplitude-modulated at a frequency Ω is incident on an inhomogeneous transparent layer of a current-carrying magneto-active plasma (ionosphere). The nonlinearity of the ionosphere leads to the appearance of currents at the difference frequency Ω. These currents produce low-frequency radiation in the earth-ionosphere waveguide. Thus, the problem is divided into two stages: the first is to determine the nonlinear currents in the ionospheric plasma, and the second is to analyze the excitation of the ground waveguide by the given currents.

When the ionosphere is irradiated by a high-power radio wave, the main contribution to the nonlinear current is made by the quadratic nonlinearity due to the change of the electron temperature in the field of the electromagnetic wave

$$\Delta T = \frac{1}{3}\,\frac{e^2}{m}\,\frac{(\Omega + 2i\nu_e)\,E_{10}\,E_{20}\exp\{i\Omega t - i(\mathbf{k}_1 - \mathbf{k}_2)\mathbf{r}\}}{(\Omega + i\delta\nu_e)\,[(\omega + \omega_{He})^2 + \nu_e^2]} + \text{c.c.} \qquad (1)$$

where ν_e is the frequency of the collisions between the electrons and the neutrals or the ions, ω_{He} is the electron gyrofrequency, δ is the fraction of the energy transferred to the electron by the heavy particles in the collision, E_{10} and E_{20} are the amplitudes of the incident-radiation components at the frequencies ω and $\omega - \Omega$, $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors, and e and m are the charge and mass of the electron. Expression (1) is valid so long as $E_0^2/E_p^2 \ll 1$ ($E_p = [3mT_e\delta(\omega^2 + \nu^2)/e^2]^{1/2}$ is the plasma field) and the self-action of the waves at the high frequency can be disregarded.

The temperature modulation leads to a change of the electron velocity at the frequency Ω, due to variations of the pressure force and the friction force. Using for the electrons the equations of motion averaged over the high frequency, it is easy to obtain the volume density of the nonlinear current at the difference frequency Ω:

$$\mathbf{J}_\Omega = \hat{\sigma}^e(\Omega)\left[\frac{m}{e}\left(\frac{\partial \nu_e}{\partial T}\right)\mathbf{U}_0 + \frac{\nabla n_0}{n_0 e}\right]\Delta T_e, \qquad (2)$$

where $\hat{\sigma}^e$ is the electronic part of the conductivity tensor of the magnetoactive plasma, $\mathbf{U}_0 = \mathbf{U}_e - \mathbf{V}_0$ ($\mathbf{U}_e = \{\mathbf{V}_0 - \beta_e[\mathbf{V}_0\times\mathbf{h}] + \beta_e^2\mathbf{h}(\mathbf{V}_0\cdot\mathbf{h})\}/(1+\beta_e^2)$, $\beta_e = \omega_{He}/\nu_e$, $\mathbf{V}_0$ is the velocity of the neutrals or of the electric drift, $\mathbf{h}$ is the unit vector along $\mathbf{H}_0$), ΔT_e is defined by (1), and n_0 is the unperturbed electron density in the ionosphere.

Figure 1 shows the altitude profiles of the horizontal current $J_{\perp\Omega}$ connected with modulation of the friction force (solid curve) and of the vertical current $J_{\parallel\Omega}$ (along $\mathbf{H}_0$) due to the pressure gradient, under the conditions of daytime medium-latitude ionosphere. The figure shows the most typical singularities, namely a sharp maximum of $J_{\perp\Omega}$ at altitudes 80–100 km and an increase of the currents with decreasing frequency.

The shift of the maximum of $J_{\perp\Omega}$ at lower altitudes relative to the maximum I_0 of the ionospheric current jet is due to the fact that $J_{\perp\Omega}$ is determined by the electronic part of the Pederson conductivity, in contrast to I_0, which is determined by the Hall conductivity.

For rough estimates of the effectiveness with which the earth-ionosphere waveguide is excited in the whistler-wave band $\Omega/2\pi \approx 10$ kHz, we can use the model of a waveguide with abrupt boundaries, one which is the high-conductivity ground (conductivity $\sigma \to \infty$), and the other is the lower edge of the anisotropic ionosphere, located at an altitude $z_0 \approx 70$ km. The excitation of the waveguide by a horizontally distributed current immersed in the ionospheric plasma is then de-

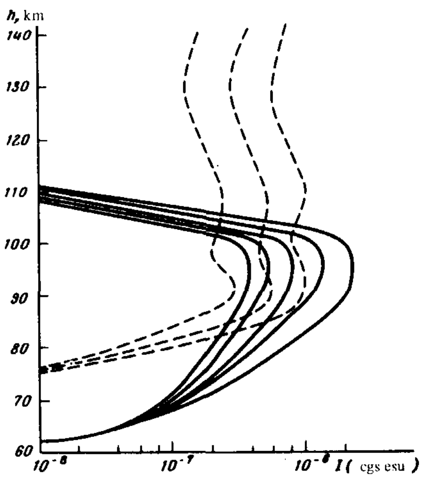

FIG. 1. Altitude dependences of $J_{\perp\Omega}$ (solid curves, frequencies 7.2, 6.2, 4, 2.5, and 1.8 kHz) and $J_{\parallel\Omega}$ (dashed, frequencies 7.2, 2.5, and 1.8 kHz).

scribed by the expression (detailed calculations are given in[2]):

$$h_\Omega = \int_v dv \frac{\pi \Omega^2 J_{\perp\Omega}}{z_0 c^3 N_i} \sum_{n=0}^{\infty} (-1)^n \delta_n (C_n) \frac{\partial \mathcal{H}_1^{(1)}(\kappa \Delta\rho S_n)}{\partial(\kappa \Delta \rho S_n)} e^{-i\kappa_i |z'-z_0|} \cos\phi, \quad (3)$$

where h_Ω is the azimuthal component of the magnetic field in the low-frequency wave on the earth's surface, $N_i = (\omega_{0e}^2/\omega_{He}\Omega)^{1/2}$ is the refractive index of the whistler wave, $\omega_{0e}^2 = 4\pi e^2 n_0/m$, the summation is carried out over the natural modes of the waveguide, $2C_n = n\pi/\kappa z_0 + [(n\pi/\kappa z_0)^2 + 4i/N_i \kappa z_0]^{1/2}$, $S_n = \sqrt{1 - C_n^2}$, z_0 is the height of the waveguide, $\kappa = \Omega/c$, $\delta_0 \approx 1$, $\delta_{n\neq 0} \approx 2$, $\Delta\rho = |\rho - \rho'|$, $dv = \rho'\, d\rho'\, d\phi'\, dz'$ is the source volume element, $\mathcal{H}_1^{(1)}$ is a Hankel function of the first kind, and ϕ is the azimuthal angle. Taking into account the dimension of the coherence of $J_{\perp\Omega}$ in the transverse direction (along $\Delta\rho$) and the asymptotic form of the Hankel function at large distances, we can approximately rewrite (3) in the wave zone in the form ($L_\perp \kappa \ll 1$, $\kappa \Delta\rho S_n \gg 1$)

$$h_\Omega = \sum_n \frac{\pi^2 \Omega^2}{z_0 c^3} L_\perp^2 (-1)^n \delta_n \cos\phi \sqrt{\frac{2}{\pi}} \frac{\exp\{i\kappa\Delta\rho S_n - i\pi/4\}}{(\kappa\Delta\rho S_n)^{1/2}} \times \int_{z_0}^{\infty} \frac{J_{\perp\Omega} e^{-i\kappa_i |z'-z_0|}}{N_i} dz', \quad (4)$$

where $L_\perp$ is the transverse radius of $J_{\perp\Omega}$ and is determined by the directivity pattern of the transmitter.

At frequencies $f < f^I = c/2z_0$ (cutoff frequencies of the first mode), S_n is a complex quantity at $n \neq 0$, and the field decreases exponentially with the distance ρ for all but the zeroth mode. As a result, on going through the frequency $f = f^I$ the field amplitude in the far zone decreases jumpwise by a factor of three. Inasmuch as $f \approx 2$ kHz under the conditions of the ionosphere, the indicated singularity explains the experimentally-observed[1] "collapse" of the amplitude of the low-frequency waves at $f < 2$ kHz. It is also easy to explain the noontime maximum of the intensity, which is connected with the appreciable intensification of the ionosphere currents at altitudes 80—90 km during the daytime hours.

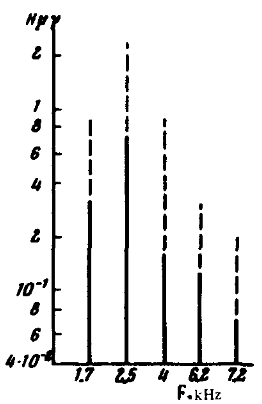

FIG. 2. Field spectrum at combination frequencies (solid curves—experiment, dashed—theory).

Estimates of the integral in (4) for different models of $N_i(z)$ and $J_{\perp\Omega}(z)$ lead to the conclusion that the main contribution to the excitation of the waveguide is made by the transition region from the D layer to the E layer (altitudes 80—90 km), where the characteristic scale a of the current variation in the vertical direction is of the order of the wavelength of the low-frequency radiation in the ionosphere, $\kappa_i a \approx 1$. With this taken into account, the order of magnitude of the integral in (4) is

$$\int_{z_0}^{\infty} (J_{\perp\Omega}/N_i) \exp\{i\kappa_i |z'-z_0|\, dz' \approx (J_{\perp\Omega}\omega_{He} c/\omega_{0e}^2)_{z\approx 8.5\ \text{km}}.$$

Under the conditions of the experiment we have $L_\perp \approx 20$ km, $\Delta\rho \sim 200$ km, $n_0 \approx 10^4$ el/cm^3, and $E_{10(20)} \approx 0.3$ V/m. Substituting these values in (4) and taking (5) and Fig. 1 into account we obtain $E_\Omega \approx 0.2$ μV/m at $f = 2.5$ kHz, in satisfactory agreement with the experimental data.

The contribution of the vertical currents to the excitation of the waveguide is smaller: in the D layer it is smaller because of the smaller amplitude of $J_{\parallel\Omega}$ in comparison with $J_{\perp\Omega}$, and in the E and F layers it is smaller because of the significant influence of the polarization effects (the low-frequency field amplitude contains the scalar product of the wave polarization vector a and the current J,[1)] a product proportional to $\Omega/\omega_{He} \ll 1$ in the E layer and above it).

Thus, the effect of nonlinear detection in the ionosphere, which was observed in[1], is due to modulation of the ionosphere currents at altitudes $h \sim 80$—90 km, and contains important information on the parameters of the lower layers of the ionosphere. One can expect an appreciable increase of the effects in the polar regions, where the intensity of the current jet is higher by two or three orders of magnitude.

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[1)]It must be borne in mind that the damping in the D layer can decrease the presented estimate of E_Ω by several times.

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[2]N. S. Belyustin, V. P. Dokuchaev, S. V. Polyakov, and V. V. Tamoĭkin, Izv. Vuzov, Radiofizika 18, 4 (1975).