Pion condensation in nuclei and l-forbidden $M1$ transitions

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A possibility is indicated of explaining the enhancement of the l-forbidden $M1$ transitions on account of the presence of pion condensation in the nuclei.

The problem of pion condensation in nuclear matter has been under intensive discussion in recent years. Interest in this question was raised by the publication of a number of papers by A.B. Migdal,[1] who was the first to develop a realist approach to this problem, based on the Fermi-liquid theory. The possibility of pion condensation in neutron matter was considered independently by Scalapino[2] and by Sawyer.[3]

In one of the first papers of this series, Migdal pointed out the feasibility of pion condensation in atomic nuclei.

The most important effect of the pion condensate on nuclear structure is an appreciable change in the form and magnitude of the tensor forces that are caused by one-pion exchange. The tensor forces determine in essential fashion the probabilities of the l-forbidden γ and β transitions. We present in this article the results of an analysis[3] of l-forbidden $M1$ transitions. In our opinion, the most natural explanation of the enhancement of transitions of the $s_{1/2}-d_{3/2}$ type lies in the existence of the pion condensate.

The reduced probability of the l-forbidden $M1$ transition is proportional to the square $|(v_2)_{if}|^2$ of the matrix element of the component $v_2(r)$ of the effective field

$$V[\sigma_\alpha] = v_1(r)\sigma_\alpha + v_2(r)(\vec{\sigma}\mathbf{n})n_\alpha , \quad (\mathbf{n} = \mathbf{r}/r) \tag{1}$$

$V[\sigma_\alpha]$ is determined by the known equation of the theory of finite Fermi systems,[4] the symbolic form of which is

$$V = V^0 - (F_0 + F_\pi)\Phi V. \tag{2}$$

Here $V^0[\sigma_\alpha] = (1-\xi_s)\sigma_\alpha$, Φ is dimensionless propagator of the particle and hole, $F_0 = (g + g'\tau_1\tau_2)\sigma_1\cdot\sigma_2$, ($h = m = c = 1$), and

$$F_\pi = \frac{1.4(1-2\xi_s)^2 \tau_1 \tau_2 (\vec{\sigma}_1 \mathbf{k})(\vec{\sigma}_2 \mathbf{k})}{1 + k^2 - \frac{0.9(1-\alpha)k^2}{1+0.23k^2} - 2\omega^2_{min}}, \tag{3}$$

where ξ_s, g, g', and α are the constants of the theory of finite Fermi systems, ω^2_{min} is the minimum of the quantity

$$\omega^2(k^2) = 1 + k^2 - \frac{0.9(1-\alpha)k^2}{1+0.23k^2} - \frac{2.8(1-2\xi_s)^2 k^2 \Phi(0,k)}{1+2g'\Phi(0,k)} \tag{4}$$

F_π describes the one-pion interaction in the annihilation channel with allowance for the virtual production of the Δ_{33} isobar and for the scattering by the pion condensate.[1]

Formula (3) for F_π holds true if the constants g', ξ_s, and α do not satisfy the Pomeranchuk stability condition, i.e., if the right-hand side of (4) becomes negative. The appearance of the pion condensate is connected precisely with this instability. If g', ξ_s, and α satisfy the stability condition, then there is no pion condensate.

In this case it is necessary to put $\omega^2_{min} = 0$ in (3).

The purest cases for analysis, in which the values of $B(M1)$ have been measured, are the transitions $2d_{3/2} - 3s_{1/2}$ in the Tl^{207} nucleus and $2f_{7/2} - 1h_{9/2}$ in Bi^{209}. In Tl we have $B(M1) = 26.2 \times 10^{-3}$, and in Bi we have $B(M1) = (4.3 \pm 0.7) \times 10^{-3}$ in nuclear magnetons squares. Both quantities (and particularly the first) is noticeably larger than the results of calculations without allowance for F_π. A term $\kappa r^2 (\sigma \cdot n) n_\alpha$, where $\kappa = 0.026$ F^{-2}, was introduced in[4] to reconcile the theory with experiment, in a calculation carried out within the framework of the standard theory of finite Fermi surface, into the bare field V^0.

Since $\langle r^2 \rangle \sim \langle R^2 \rangle \sim 50$ F^2, introduction of such a term makes the transition actually allowed and explains the experimental results. However, introduction of such a large term in V^0 contradicts the entire aggregate of experimental data on the magnetic moment and on $M1$ transitions. In addition, this term is not translationally invariant.

We have calculated $B(M1)$ for the indicated nuclei, solving Eq. (2) in the region where the constants are stable. The table lists the results of the calculation for the constants $\xi_s = 0.1$ and $\alpha = 0$, as well as for a number of values of $g' > 0.6$ (the stability condition for these values of α and ξ_s is violated at $g' < g^{cr} = 0.6$). We see that in both cases the discrepancies are quite appreciable. In the case of Bi^{209}, a reasonable result is obtained only for the value $g' = 0.8$, which is quite close to critical. In Tl^{207}, even at this value of g', the deviations from experiment exceed a factor of 6. The table gives the results of a calculation performed under the assumption that the pion condensate exists. We see that the calculation leads in the case of Tl^{207} to values close to the experimental one, and in the case of Bi^{209} to even noticeably higher values. The latter fact should not lead to undue confusion, since estimates show that in the case of Bi^{209} an appreciable contribution can come from tensor forces of non-one-pion origin (the tensor part of the "core"), which make practically no contribution in Tl^{207}. Allowance for these forces in the case of Bi^{209} can decrease the result noticeably. Thus, the assumption that a pion condensate exists in the nucleus explains qualitatively the experimentally observed lifting of the hindrance of the l-forbidden transitions.

Nucleus transition	No condensate $\xi_s = 0$			Experiment	Condensate exists	
					$\xi_s = 0$	$\xi_s = 0.1$
	$g' = 1.2$	$g' = 1$	$g' = 0.8$		$g' = 1.15$	$g' = 0.5$
Tl^{207} $2d_{3/2} - 3s_{1/2}$	2.7	2,45	4.1	26.2 [5]	16	18
Bi^{209} $2f_{7/2} - 1h_{9/2}$	0.07	0,32	2.4	4.3 ± 0.7 [5]	9.2	19
Pb^{209} $2g_{9/2} - 1h_{11/2}$	0.001	0.001	0.04	—	0.015	0.02

Dependence of $B(M1)$ (10^{-2} nuc. magnetons squared) of l-forbidden transitions on the parameters of the theory without pion condensate and with the condensate taken into account.

We note that the calculation performed for the transition $2g_{9/2} - 1i_{11/2}$ in Pb^{209}, where there is no experimental data, leads to very small values of $B(M1)$, independently of the presence or absence of pion condensate in the nucleus. At the same time, introduction of a large maximum charge proportional to $(\sigma \cdot n) n_\alpha$ leads here, too, to a large value of $B(M1)$. Therefore, measurement of the probability of this transition would serve as an important test of the proposed explanation.

The existence of the pion condensate explains qualitatively also the enhancement of the $s_{1/2} - d_{3/2}$ transitions in nuclei of medium atomic weight; for these, however, the accuracy of the calculations is much lower, since the transitions can be essentially non-single-particle.

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