# Hadrons from leptons?

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The possibility is discussed of the appearance of strongly interacting particles as excitations of a system of weakly interacting fields.

This article discusses the possible physical consequences ensuing from a consideration of a simple model of field theory in two-dimensional space-time. The model describes the self-action of a chiral field $\chi(x) = \exp[iu(x)]$, associated with the group $U(1)$. The corresponding Lagrangian

$$L = \frac{1}{\gamma}\int_{-\infty}^{\infty}\left[\frac{1}{2}(u_t^2 - u_x^2) - (1 - \cos u)\right]dx$$

expressed in a system of units $h = m = c = 1$, where $m$ is the mass of the field $u$, contains one dimensionless parameter $\gamma$, which plays the role of a coupling constant. In[1], the inverse-problem method was used to obtain a complete description of the classical equations of motion for this Lagrangian, and an expression was obtained for the Hamiltonian in terms of variables of the "particle number" and "phase" type. It follows from these results in the semiclassical approximation that when the foregoing Lagrangian is quantized the result is an entire spectrum of particles; in addition to the mass-1 particle corresponding to the weak-field approximation, this spectrum contains the following: (a) a particle of mass $8/\gamma$, which has a charge $\pm 1$ associated with a trivially-conserved current $I_\mu = (1/2\pi)\epsilon_{\mu\nu}\partial_\nu u$ (soliton); (b) a series of particles with zero charge and with masses $M_n = (16/\gamma)\sin\theta_n$, where $\theta_n = (\pi/4N)(2n+1)$, $n = 0, \ldots, N-1$; $N = [8\pi/\gamma]$ (bound states of solitons). A typical solution of the problem of classical soliton scattering[2] makes it also possible to calculate the quasiclassical approximation for the corresponding matrix elements of the $S$ matrix. For example, for two solitons we have

$$S(s) = \exp\left\{\frac{16i}{\gamma}\int_1^{\xi}\frac{dx}{x}\ln\frac{x-1}{x+1}\right\}; \quad \xi = \frac{s - 2m^2 + \sqrt{s(s-4m^2)}}{2m^2}; \quad m = \frac{8}{\gamma}.$$

A more accurate quasiclassical approximation can be obtained, for example, with the aid of continual integration. It turns out in this case that the quantum corrections to the mass and phase shift of the soliton scattering are represented by a series in nonnegative powers of $\gamma$. In other words, at small values of $\gamma$ the quasiclassical results are qualitatively correct and show, in particular, that the soliton masses can be made large and their interaction can be made strong.

Thus, the example considered shows that there exists a mechanism capable of describing also strongly-interacting particles as collective excitations in a system of weakly-interacting fields. The new particles then have whole-number characteristics (such as charge) that are equal to zero for particles corresponding to the initial fields in perturbation theory. This mechanism is attractive to the highest degree. Indeed, let us imagine that we have full faith in quantum field theory as the fundamental basis of elementary-particle theory. Then there should exist a Lagrangian containing several fundamental fields and such that particle-like excitations corresponding to these fields describe the entire spectroscopy of the observed particles and the hierarchy of their interactions. It would be extremely ugly if the fundamental fields were to include spinor fields for both leptons and baryons (quarks), so that the number of fields of like type is doubled. The described mechanisms gives grounds for hoping that it suffices to choose as fundamental the fields of the leptons and the vector fields that transfer their interactions. The hadrons should appear in this case as solitons or their bound states, with the soliton charge playing the role of the baryon number.

Let us see now which features of the model system should be preserved in the real case of four-dimensional space-time in order for this mechanism to be operative. First, the corresponding classical equations of motion should have solutions of the soliton type, i.e., localized finite-energy solutions that are stationary or periodic in time. Second, there should exist a conserved current such that the soliton solutions have a nontrivial corresponding charge that vanishes in the case of weak fields. This condition is satisfied if the fundamental fields include a chiral field connected, for example, with the $SO(3)$ group or with the homogeneous space $SO(3)/SO(2)$. Thus, in the former case the conserved current is of the form $I_\mu = (2\pi^2)^{-1}(1+u^2)^{-3}\epsilon_{\mu\nu\lambda\sigma}\epsilon^{abc}\partial_\nu u^a\partial_\lambda u^b\partial_\sigma u^c$, where we are using the Weinberg parametrization for the chiral field. We note that both forementioned currents are conserved regardless of the form of the equations of motion, and the whole-number character of the corresponding charges is connected with their topological meaning—these charges determine the so-called numbers of image rotations realized by the chiral fields (cf.[3–5]). The whole-number conservation laws that appear in[6–8] and are connected with the nontrivial asymptotic form of the considered stationary solutions at infinity of space have a similar but not identical interpretation. In particular, in the two-dimensional case[6] the produced charge does not have the additivity property.

The two conditions of the foregoing paragraph are realized in the unified model proposed in[9] for electromagnetic and weak interactions of leptons. The fields of the model include, besides the triplet of leptons and the triplet of Yang-Mills fields (the electromagnetic field and charged intermediate bosons) also the chiral field of directions $n = (n_1, n_2, n_3)$, $n_1^2 + n_2^2 + n_3^2 = 1$, which has the geometric meaning of an electrically neutral direction in the internal lepton space. It is shown in[10] that in the case of three-dimensional space-time the classical equations of motion of the model have stationary solutions with finite energy and nontrivial topological charge. The energy of these solutions is inversely proportional to the weak-interaction constant. In the real four-dimensional case, the existence of such solutions implies the existence of solutions that are concentrated about a finite closed contour (string) that executes periodic nonlinear oscillations.[11,12] This motion should lead, upon quantization, to an infinite series of discrete values for the mass of the corresponding excitations. All this shows that in lepton theory one can have excitations that have large masses, nontrivial quantum numbers, and strong interaction. A comparison of their properties with real hadrons can be carried out only after we learn how to solve the problem of quantization of a closed string in four-dimensional space.


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