Possible interpretation of the maximum plasma pressure in a stellarator

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We demonstrate the possibility of splitting the magnetic configuration of the stellarator into resonant structures in the presence of a plasma. This fact can lead to a limitation on the maximum plasma pressure in the trap.

It is known that the integral characteristics (dimensions and shapes of the magnetic surfaces, the shear and the angle of the rotational transformation of the force lines, the "magnetic well") determine in practice the instability of the magnetic structure, and also the equilibrium and stability of plasma in closed traps. However, with increasing gas kinetic pressures of the plasma, these characteristics of the magnetic systems of the stellarator type can be appreciably altered.[1,2]

Calculations carried out within the framework of the method of averaging[3] the differential equation of the force lines make it possible, in first-order approximation in β and δ, to write down the equation of the magnetic surfaces, for an $l=3$ toroidal stellarator, in the form[1)]

$$\bar{\Psi} = \bar{\Psi}_0 + \beta \frac{m\delta}{192\lambda^2\epsilon^2 r_0^2 (m-2)}\left[\frac{1}{1-\bar{r}_0^{m+2}} - \frac{4}{m+a}\left(\frac{\bar{r}}{\bar{r}_0}\right)^{m-2}\right] r\cos\theta$$

$$\bar{\Psi}_0 = \lambda^2\left\{6\bar{r}^4 - \delta\left[\frac{24}{5}\bar{r}^5 + 3\bar{r}^3(1+2\bar{r}^2)\cos\theta\right]\right\}, \quad (1)$$

where $\bar{r}$ is the distance from the central line of the torus in units of a; θ is the polar angle in the meridional plane and is measured from the principal normal to the torus axis; $\delta = a/R$ (a and R are the minor and major radii of the torus); $\beta = P_0/(B_0^2/8\pi)$ (P_0 is the gas-kinetic pressure of the plasma on the central line of the torus); $\epsilon = l\delta$ (l is the number of turns of the current pole around the torus); $\lambda = 2I/caB_0\epsilon$ (I is the current at the

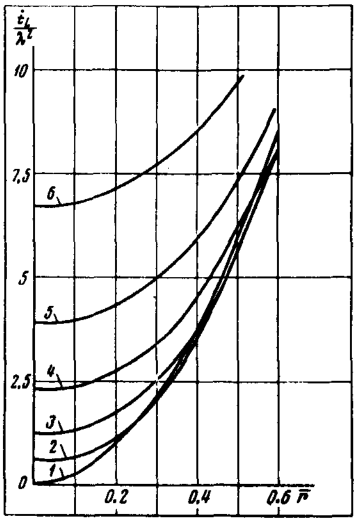

FIG. 2.

pole of the helical lining of the stellarator, B_0 is the external longitudinal magnetic field); $P(\bar{r}) = P_0[1-(\bar{r}/\bar{r}_0)^m]$ (m is the degree of the parabola and $\bar{r}_0$ is the average radius of the plasma in units of a).

The angle of rotational transformation relative to the displaced magnetic axis $(\bar{r}_c, \bar{\theta}_c)$ in an $l=3$ system is determined by the expression

$$i_L = 2\pi \dot{i}_L = 2\pi A \Big/ \left(1 + \frac{1}{2}\mu^2 + \frac{1}{2}\chi^2 - \frac{3}{4}\mu^2\chi\right); \qquad (2)$$

$$\mu = \frac{B}{A}; \qquad \chi = \frac{C}{A},$$

where

$$A = \lambda^2[24\bar{r}_1^2 + 12\bar{r}_c(4\bar{r}_c - \gamma\delta)] - \Delta\gamma\frac{8\bar{r}_c}{3\bar{r}_0^4};$$

$$B = \bar{r}_1\left[9\lambda^2(8\bar{r}_c\gamma - \delta) - \frac{2\Delta}{\bar{r}_0^4}\right]; \quad C = 6\lambda^2\bar{r}_c(4\bar{r}_c - \gamma\delta) - \Delta\gamma\frac{4\bar{r}_c}{3\bar{r}_0^4};$$

$$\Delta = -\gamma\lambda^2\bar{r}_0^2\frac{24\bar{r}_c^3 - \delta[24\bar{r}_c^4 + 3\bar{r}_c^2\gamma(3+10\bar{r}_c^2)]}{1-2(\bar{r}_c/\bar{r}_0)^2}; \qquad \gamma = \cos\theta_c$$

$\bar{r}_1$ and $\bar{\theta}_1$ are the coordinates relative to the displaced axis. The figure shows the radial dependence of the angle of rotational transformation of the force lines, $i_L/\lambda^2 = f(\bar{r}_1)$, as determined by expression (2) for different values of the displacement of the magnetic axis at $\delta = 0.1$, $\bar{r}_0 = 0.6$ ($1-\bar{r}_c=0$; $2-\bar{r}_c=0.1$; $3-\bar{r}_c=0.15$; $4-\bar{r}_c=0.2$; $5-\bar{r}_c=0.25$; $6-\bar{r}_c=0.3$).

An analysis of (1) shows that in the presence of the plasma the internal magnetic surfaces are shifted to the outside of the torus much more strongly than the peripheral ones. This realignment of the surfaces causes significant changes in the dependence of the rotational-conversion angle on the average radius $\bar{r}_1$. It follows from (2) and from the figure that increasing the plasma pressure gives rise to an angle of rotational conversion on the axis and to a decrease of the shear in the central region of the configuration.

	P	$\bar{r}^*$	$\bar{r}_v^*$	S	S_v	$\tilde{\rho}/\bar{r}^*$	$\tilde{\rho}_v/\bar{r}_v^*$	$\tilde{\rho}$	$\tilde{\rho}_v$
$\beta=5\cdot10^{-3}$; $\bar{r}_c=0.25$	3	0.3	0.46	$4.8\cdot10^{-3}$	$3.2\cdot10^{-2}$	0.58	0.222	0.174	0.1
$\beta=10^{-2}$; $\bar{r}_c=0.3$	2	0.25	0.56	$2.4\cdot10^{-3}$	$6\cdot10^{-2}$	1	0.2	0.25	0.112

It is known[5-7] that at rational values of the rotational-conversion angle and in the presence of small extraneous perturbations a small value of the shear (and of its derivative with respect to the radius) can lead to a splitting of the magnetic surfaces into a number of rosettes, the relative widths of which can be estimated from the expression[5,7]

$$\frac{\tilde{\rho}}{\bar{r}^*} = 4\sqrt{\tilde{B}/B_0 PS}, \qquad (3)$$

where B/B_0 is the relative amplitude of the perturbation; P is the number of turns of the force line along the torus (the number of the resonance); S is the shear near the rational surface $\bar{r}^*$.

By way of example, the table[2)] lists the results of estimates of the rosette dimensions for the particular case of an $l=3$ stellarator with parameters $1/\lambda = 9.43$, $\delta = 0.1$, $l=6$, and $B/B_0 = 3\times10^{-4}$. $\bar{r}_v^*$, S_v, and $\tilde{\rho}_v$ are the vacuum values of the corresponding quantities.

As seen from the table, with increasing pressure an $l=3$ stellarator loses the advantage created by the large value of the vacuum shear. It is known from experimental data[6,7] that the appearance of rosettes with relatively large dimensions leads to a strong drift of the plasma across the magnetic field. It should be noted that analogous phenomena (the decrease of the shear and the increase of the rotational-conversion angle on the axis) can occur in the presence of a plasma also in other stellarator systems. In particular, if the vacuum angular characteristics of an $l=2$ stellarator lie far from the principal resonances, then these characteristics can assume resonant values with increasing β, and can lead in the presence of small perturbations to degeneracy of the surfaces.

An analysis of these results allows us to assume that the limitation of β in stellarators is the result of the splitting of the magnetic configuration into resonant rosettes that are connected with the decrease of the value of the shear and with the increase of the angle of rotational conversion on the system axis.

1)We use the magnetic-field components obtained in[4] for an $l=3$ toroidal stellarator.

2)The amplitudes of the perturbations, as indicated in[7], lie in the interval $10^{-4}-10^{-3}$ for the fundamental resonances, and conform to the precision with which the stellarator is constructed.

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