Contribution to the microscopic theory of the Josephson effect in superconducting bridges

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(Submitted December 22, 1974)
ZhETF Pis. Red. **21**, No. 4, 216–219 (February 20, 1975)

We consider the properties of superconducting bridges (S–C–S junctions) in the dirty limit $l < \xi_0$ at arbitrary temperatures. We find the dependence of the Josephson current on the phase shift. At $T=0$ we have $dI/d\phi = -\infty$ at the point $\phi = \pi$.

Superconducting junctions of the constricting type (S-C-S), such as bridges and point junctions, are presently under diligent experimental study, and seem to be of greatest interest for practical applications. At the same time, in contrast to S-I-S junctions,[1] they have not been studied theoretically to a sufficient degree. They have been analyzed so far within the framework of the Ginzburg-Landau theory, i.e., at temperatures close to T_c,[2] where the properties of the bridges coincide in the main with the properties of Josephson tunnel junctions, although there are some interesting differences (see, e.g.,[3]). In this paper we investigate S-C-S junctions at arbitrary temperatures on the basis of microscopic equations.

We consider a weak superconducting junction in the form of a filament of length L and diameter a, joining two bulky superconducting half-spaces (for the sake of argument, we consider a filament, but the entire discussion that follows pertains also to film bridges of variable thickness[4]). If $a \ll L$ and $a \ll \xi$, we can solve the one-dimensional problem with boundary conditions specified by the superconducting "edges," neglecting the change of the order parameter in bulky superconductors under the influence of the flowing current; ξ is the coherence length, $\xi(T) = \xi(0)/\sqrt{1-T/T_c}$, where $\xi(0) = \sqrt{\xi_0 l}$ (l is the mean free path). We consider the case of a dirty superconductor, $l \ll \xi_0$ and $a > l$.

In the dirty limit, we can use for the order parameter Usadel's equations,[5] which follow from the quasiclassical (in terms of the parameter λ_F/ξ_0) equations of the Eilenberger superconductivity theory[6,7] at $l \ll \xi_0$. In the one-dimensional case these equations take the form

$$2\omega F(\omega, x) - D\frac{d}{dx}\left\{(1-|F|^2)^{1/2}\frac{dF}{dx} + \frac{1}{2}\frac{F}{(1-|F|^2)^{1/2}}\frac{d|F|^2}{dx}\right\} = 2\Delta(x)[1-|F|^2]^{1/2}, \tag{1}$$

$$\Delta(x) = \lambda N(0)\pi T\sum_\omega F(\omega, x). \tag{2}$$

$F(\omega, x)$ is Gor'kov function integrated over the energies, $\Delta(x)$ is the order parameter, $\omega = (2n+1)\pi T$, and $D = (1/3)v_F l$.

The current density is expressed in terms of $F(\omega, x)$ by the relation

$$j_s = -eiN(0)\pi TD\sum_\omega\left(F^*\frac{dF}{dx} - F\frac{dF^*}{dx}\right). \tag{3}$$

We solve these equations in the interval $x \in [-L/2, L/2]$ with boundary conditions for $\Delta(x)$ and $F(\omega, x)$ at $x = \pm L/2$;

$$\Delta(\pm L/2) = \Delta_0 e^{\pm i\phi/2}, \qquad F(\omega, x=\pm L/2) = \frac{\Delta_0}{\sqrt{\omega^2+\Delta_0^2}}e^{\pm i\phi/2} \tag{4}$$

ϕ is the phase difference between the edges of the bridge, and $F_0 = \Delta_0/\sqrt{\omega^2+\Delta_0^2}$ is the homogeneous solution in the bulky "edges."

The case of a weak junction corresponds to the limit $L \ll \xi$, in which case one can neglect in (1) all the terms except the gradients. The obtained equations with the

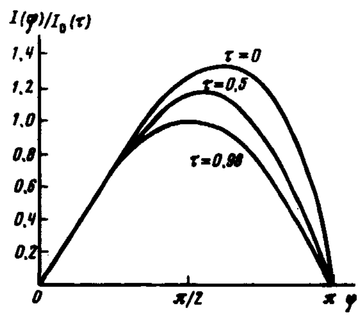

FIG. 1.

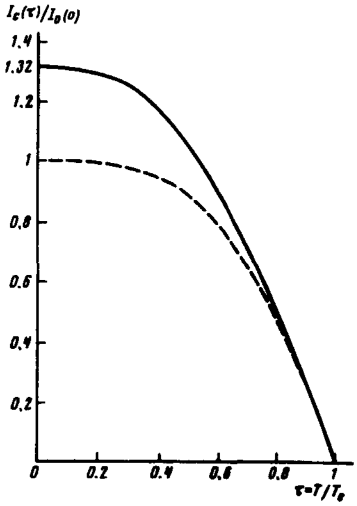

FIG. 2.

boundary condition (4) can be easily solved. Substituting the obtained solution in expression (3) for the current (which for convenience is calculated at the point $x=0$), we find that the total current through the bridge, at a phase difference ϕ between the "edges," is

$$I = \frac{2\Delta_0}{eR_N}\cos\frac{\phi}{2}\,\pi T\sum_{\omega}\frac{1}{\sqrt{\omega^2+\Delta_0^2\cos^2\phi/2}}\,\mathrm{arc\,tg}\frac{\Delta_0\sin\phi/2}{\sqrt{\omega^2+\Delta_0^2\cos^2\phi/2}}. \quad (5)$$

R_N is the resistance of the filament in the normal state, and Δ_0 is the modulus of the order parameter at the edges in the absence of current, at a given temperature. An analysis shows that relation (5) between the current and the phase shift remains valid also when a hyperbolic model is used for the bridge,[2] i.e., in the weak-coupling limit this relation is insensitive to the shape of the junction.

At $T\sim T_c$ we have $I=(\pi\Delta_0^2/4eR_NT_c)\sin\phi$. At arbitrary temperatures, the current is a periodic function of the phase shift, with period 2π; this relation is shown for different temperatures ($\tau=T/T_c$) in Fig. 1. $I_0(\tau) = (\pi\Delta_0/2eR_N)\tanh(\Delta_0/2T)$ is the critical current corresponding to the Ambegoakar-Baratov formula (see[1]).

At $T=0$, changing in (5) from summation to integration, we obtain $(-\pi\leqslant\phi\leqslant\pi)$

$$I = \frac{\pi\Delta_0}{eR_N}\cos\frac{\phi}{2}\,\mathrm{Ar\,th}\sin\frac{\phi}{2} = \frac{\pi\Delta_0}{2eR_N}\sin\phi\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\sin\frac{\phi}{2}\right)^{2n}. \quad (6)$$

A characteristic feature of the phase dependence of the current $I(\phi)$ is that the derivative $(dI/d\phi)_{\phi=\pi}$ goes to $-\infty$ at $T=0$. We note that a similar behavior of $I(\phi)$ is typical also of S-N-S junctions.[8] At low temperatures $T\ll T_c$ we have asymptotically $(dI/d\phi)_{\phi=\pi}\approx I_0\ln(T/T_c)$. The critical current at $T=0$ is equal to $1.32I_0$, where $I_0=\pi\Delta_0/2eR_N$. A plot of the critical current $I_c(T)$ at arbitrary temperature, is shown in Fig. 2 (the dashed line is a plot of $I_0(T)$).

We have developed in this paper a theory for the Josephson effect in short bridges $L<\xi$. With increasing length of the bridge, the Josephson behavior gives way to that of a long filament. In the Josephson model, the critical length of the bridge at $T\sim T_c$ is $L_c\approx 3.65\xi$,[4] and at low temperatures the increase of the slope of the $I(\phi)$ curve at $\phi=\pi$ leads to a decrease in the ratio L_0/ξ. As $T\to 0$ we get $L_c\sim\xi/\ln(T_c/T)$. Our analysis is valid at $L<L_c$.

In conclusion, we are grateful to K.K. Likharev for a discussion of the work and to É.A. Kel'man for help with the numerical calculations.

[1]I.O. Kulik and I.K. Yanson. Éffekt Dzhozefsona v sverkhprovodyashchikh tunnel'nykh strukturakh (Josephson Effect in Superconducting Tunnel Structures), Nauka, 1970.
[2]L.G. Aslamazov and A.I. Larkin, ZhETF Pis. Red. **9**, 150 (1969) [JETP Lett. **9**, 87 (1969)].
[3]K.K. Likharev, Zh. Eksp. Teor. Fiz. **61**, 1700 (1971) [Sov. Phys.-JETP **34**, 906 (1972)].
[4]K.K. Likharev, L.A. Yakobson, and G.M. Lapir, Tezisy dokladov na 18 Vsesoyuznom sobeshchanii po fizike nizkikh tempteratur (Abstract of Papers at 18th All-Union Conference on Low-Temperature Physics), Kiev, 1974, p. 227.
[5]K.D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[6]G. Eilenberger, Z. Physik **214**, 195 (1968).
[7]A.I. Larkin and Yu.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **55**, 2262 (1968) [Sov. Phys.-JETP **28**, 1200 (1969)].
[8]I.O. Kulik, Slabaya sverkhprovodimost' (Weak Superconductivity) Preprint, Institute of Metal Physics, Siberian Division, USSR Academy of Sciences, Sverdlovsk, 1973.