Supergauge renormalizable theory of massive vector field

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We consider a supergauge theory, in which an Abelian massive vector field interacts with a nonconserved current. The renormalizability of the unity S matrix on the mass shell is proved by a transition to a nonunitary representation of the algebra of supergauge transformations.

The supergauge theory of a massive vector field interacting with a conserved current was considered in[1,2]. In this renormalizable model, all the quadratic divergences cancel out in the mass operators of the boson fields.[1)] A hypothesis was therefore advanced, according to which the group of supergauge transformations can be used to extend the class of renormalizable theories. In this paper we consider a renormalizable supergauge model which turns out to be renormalizable as a result of cancellation of divergences on the mass shell.

The algebra of the generators of the supergauge transformations takes the following form[1,2]:

$$[W, \bar{W}]_+ = \overset{+}{\gamma}_\mu P_\mu, \quad [W, W]_+ = [\bar{W}, \bar{W}]_+ = 0, \qquad [P_\mu, W]_- = [P_\mu, \bar{W}]_- = 0, \tag{1a}$$

$$\bar{W} = W^+ \gamma_0, \tag{1b}$$

where P_μ is the generator of the four-dimensional translations, W and $\bar{W}$ are the generators of the spinor translations,[2)] $\overset{\pm}{\gamma}_\mu = \overset{\pm}{S}\gamma_\mu$, $\overset{\pm}{S} = (1 \pm \gamma_5)/2$, and $\gamma_5^2 = 1$. By way of example of a nonrenormalizable theory within the framework of the algebra of[1] we consider a model in which a massive vector field interacts with a nonconserved current. Such a model is obtained by considering the self-action of the multiplet consisting of a scalar, spinor, and vector fields. The simplest superinvariant interaction of these fields is described by the following Lagrangian:

$$\begin{aligned} L &= \int L_0 d^3x + g\int L_1 d^3x + g^2 \int L_2 d^3x, \\ L_0 &= \tfrac{1}{2}(\partial_\alpha \chi)^2 - \tfrac{1}{2}\mu^2\chi^2 + \tfrac{i}{2}\bar{\psi}\gamma_\alpha \overleftrightarrow{\partial}_\alpha \psi - \mu\bar{\psi}\psi \\ &\quad - \tfrac{1}{4}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)^2 + \tfrac{1}{2}\mu^2 A_\alpha^2, \\ L_1 &= \bar{\psi}\tilde{\gamma}_\alpha \psi A_\alpha - \bar{\psi}\psi\chi + \mu A_\alpha^2 \chi - \tfrac{1}{2}\mu\chi^3, \\ L_2 &= \tfrac{1}{2}A_\alpha^2\chi^2 - \tfrac{1}{8}\chi^4. \end{aligned} \tag{2}$$

The nonlinearity of the supergauge transformations, specified by the operators W and $\overline{W}$, leads to the appearance in the Hamiltonian (and Lagrangian) of terms that are quatrilinear in the field operators; this makes it difficult to investigate the divergences of the S-matrix elements. These terms can be included by making a change of variables, such as to convert the operator W (but not $\overline{W}$) into the linear operator

$$B'_a = A_a - i\partial_a \chi(\mu + g\chi)^{-1}, \qquad \chi' = \chi + \frac{g}{2\mu}\chi^2,$$
$$\overset{+}{S}\psi' = \overset{+}{S}\psi\left(1 + \frac{g}{\mu}\chi\right), \qquad \bar{S}\psi' = \bar{S}\psi, \tag{3}$$
$$\bar{\psi}'\bar{S} = \bar{\psi}\bar{S}\left(1 + \frac{g}{\mu}\chi\right)^{-1}, \qquad \bar{\psi}'\overset{+}{S} = \bar{\psi}\overset{+}{S}.$$

We forgo here the requirements (1b), i.e., we go over to a nonunitary representation of the supergauge algebra. In this representation, the Lagrangian takes the form

$$L' = \int L'_0 d^3x + g\int L'_1 d^3x, \tag{4}$$
$$L'_0 = i/2\,\bar{\psi}'\gamma_a \overleftrightarrow{\partial_a}\psi' - \mu\bar{\psi}'\psi' - 1/2\,\mu^2(\chi')$$

$$-1/4(\partial_a B'_\beta - \partial_\beta B'_a)^2 + 1/2\,\mu^2(B'_a)^2 + i\mu\partial_a\chi' B'_a \tag{4a}$$

$$L'_1 = \bar{\psi}'\gamma_a\psi' B'_a - 2\bar{\psi}'\bar{S}\psi'\chi' + \mu(B'_a)^2\chi', \tag{4b}$$

with

$$L'_0(\chi', \psi', A'_a) = L_0(\chi', \psi', A'_a), \tag{5}$$

$$A'_a = B'_a + i/\mu\,\partial_a\chi'. \tag{6}$$

The aggregate of the transformations (3) and (6) satisfies the requirements of the equivalence theorem, and consequently does not change the renormalized S matrix on the mass shell.[5] Therefore the S matrix on the mass shell, expressed in terms of the fields χ', ψ', and A'_α, is unitary in spite of the non-Hermitian character of the interaction Lagrangian (4b).

We proceed to a determination of the divergences in the theory with a Lagrangian (4). We note that the maximum degree of the momentum in the numerator of the propagators is not higher than unity. In addition, the number of such propagators in an arbitrary diagram does not exceed the number of vertices. This means that the considered theory is renormalizable with respect to the index of the divergence. The use of the Ward identities makes it possible to investigate the structure of the divergences of the S-matrix elements and introduce the renormalized quantities (we leave off the primes from the symbols for the fields):

$$\chi^R = Z^{1/2}Z^{-1/2}\chi, \qquad B_a^R = Z_2^{-1/2}B_a$$
$$\overset{+}{S}\psi^R = Z^{1/2}Z_2^{-1/2}\overset{+}{S}\psi, \qquad \bar{\psi}^R\bar{S} = Z^{1/2}Z_2^{-1/2}\bar{\psi}\bar{S},$$
$$\bar{S}\psi^R = Z_2^{-1/2}\bar{S}\psi, \qquad \bar{\psi}^R\overset{+}{S} = Z_2^{-1/2}\bar{\psi}\overset{+}{S}. \tag{7}$$
$$\mu_R = Z^{-1/2}\mu, \qquad g_R = g Z_2^{3/2}Z_1^{-1}$$

After making the variable changes (7), the Lagrangian (4) takes the form

$$L' = Z_2\int L'_0 d^3x + Z_1 g_R\int L'_1 d^3x + (Z-1)Z_2\int \Delta L' d^3x,$$
$$\Delta L'_0 = 1/2\,(\mu_R)^2(B_a^R)^2 - \mu_R\bar{\psi}^R\bar{S}\psi^R,$$

where the arguments χ, ψ, B_α, and μ in L'_0 and L'_1 are replaced by χ^R, ψ^R, B^R_α, and μ_R.

The employed proof of renormalizability did not use the fact that the model with the Lagrangian (2) can be regarded as a spontaneously violated gauge model of special type in a unitary gauge. Thus, supersymmetry permits a new approach to the renormalizability of the theory of the massive vector field. In addition, supersymmetry leads to a larger number of relations between the coupling constants than in the gauge models. In particular, as seen from formula (2), a relation is obtained between the axial and vector coupling constants, i.e., violation of P-invariance of the theory.

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1) The connection between mass renormalization and wave-function renormalization in an analogous model was obtained in[3].

2) A classification of the extensions of the algebra of the generators of the Poincaré group by bispinor generators is given in[2,4]. Commutation relations for the Majorana spinor $Q = W + C\overline{W}^T$ in the case of the algebra of[1] have been written out in[3].

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