A new singularity in the dispersion of impurity excitation in superfluid helium

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It is shown that the impurity branch of the energy spectrum of the elementary excitations of He^3–He^4 solutions has an end point at which decay into a roton and an He^3 quasiparticle with small momentum takes place. The parameters of this spectrum are obtained.

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The difference between the conclusions based on the reduction of the experimental data on the velocity of fourth sound,[1] ion mobility,[3] and the density of the normal component,[3] on the one hand, and the results of direct measurements[4–6] of the parameters of the boson branch in solutions of He^3 in superfluid He^4 on the other hand, have increased the interest in the problem of the elementary-excitation spectrum.[7,8]

The resultant situation calls, as will be shown subsequently, for a review of the presently held opinions concerning the spectrum of the impurity excitations.

This paper is devoted to a derivation of the dispersion law of He^3 quasiparticles on the basis of the experimental data on the density ρ_n of the normal component at temperatures at which the impurity quasiparticles are subject to Boltzmann statistics.

To solve this problem we used the fact that, within the framework of the quasiparticle model, the dependence of the impurity part ρ_{ni} of the normal component on the temperature T is determined by the law governing the dispersion of the He^3 quasiparticles. The value of ρ_{ni} was determined from the formula[8]

$$\rho_{ni}/\rho = \kappa_i = \frac{\kappa - \kappa_0}{1 - \kappa_0}, \qquad (1)$$

where κ and κ_0 are respectively the total normal density and the relative density due to thermal excitations. The values of κ were taken from[3], and κ_0 was calculated with allowance for the data on the roton parameters and the smearing of the energy gap,[9] which are given in[4–6].

It is important that κ_i tends at low temperatures to a constant value $\kappa_i(0)$, and an increase of T leads to a noticeable growth of the impurity part of the normal density. Figure 1 shows a plot of $\xi(T)$, where

$$\xi = [\kappa_i(T) - \kappa_i(0)]x^{-1}, \qquad (2)$$

and x is the weight concentration. It follows from the plot that the values of ξ for solutions with different concentrations agree within the limits of errors (the errors being 0.1, 0.06, and 0.04 for $x = 0.085$, 0.156, and 0.256, respectively).

This fact allows us to conclude that the interaction between the He^3 impurities does not influence the temperature dependence of ρ_{ni}, which is connected with the type of dispersion law. In addition, at large values of the momentum, the parameters of the impurity-excitation spectra, like the parameters of rotons in solutions,[4–6] are practically independent of the concentration.

On the basis of the foregoing considerations, we can attempt to reconstruct the energy spectrum of the He^3 quasiparticles. It is necessary here, however, to start from some specified function $\epsilon(p)$ with unknown parameters and then, using the values of $\rho_n(T)$, to determine these parameters.

The first attempt of this type was connected with an idea advanced recently by L.P. Pitaeveskiĭ (see[8]).

It appears, however, that a more general approach is the one based on representing the energy ϵ in the form

$$\epsilon = p^2/2m^*(1 + \Sigma\alpha_k p^{2k}), \qquad (3)$$

where p and m^* are respectively the momentum and the effective mass of the quasiparticle.

To determine α_1 and α_2 we used the ρ_n data at $T < 0.06$ K given in[10]. At these low temperatures it is possible to expand κ_i in powers of α_1 and α_2. A least-squares reduction of the data has shown that α_1 is negligibly small and $\alpha_2 = -0.04$ Å^4.

The next coefficient, α_3, is obtained from the condition that the quantity

$$\Phi = \Sigma\left[\xi - 5.37\left(\int_0^t p^4 n dp\right)\left(T\int_0^t p^2 n dp\right)^{-1} + \frac{m^*}{m_3}\right]^2 \qquad (4)$$

be a minimum, where n is the Boltzmann function and

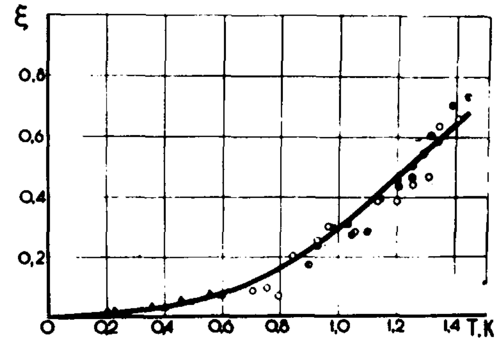

FIG. 1. Temperature dependence of ξ: the points ▲, △, ●, ○, and ◐ correspond to weight concentrations 0.005,[11] 0.010,[10] 0.85, 0.156, and 0.256.[3] The solid curve was plotted using (3).

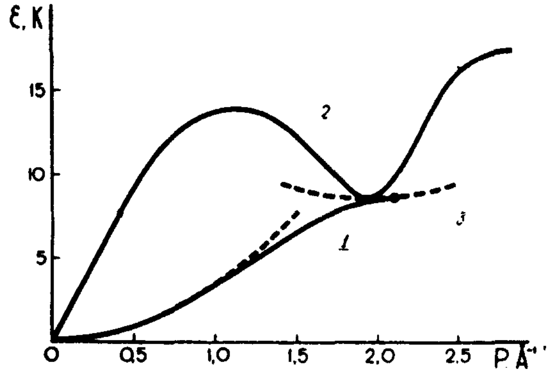

FIG. 2. Energy spectrum of elementary excitations.

the summation is carried out over the experimental points[3] corresponding to the temperature region 0.7–1.4 K.

The $\epsilon(p)$ curve obtained in this manner might cross the boson branch. However, as shown by L.P. Pitaevskiĭ,[12] the spectrum should have in this case an end point connected with the decay of the excitation. Using the results of[12], it can be shown that in this case the decay is accompanied by production of a roton and an He^3 quasiparticle with small momentum parallel to the roton momentum.

Since the conservation laws impose a connection between the energy ϵ_c and the momentum p_c at the end point of the spectrum, and the velocities of the decaying and produced excitations are equal,[12] it becomes possible, with only one varied parameter p_c, to retain the coefficients α_3 and α_4 in (3). The sought value of p_c corresponds to the minimum of Φ ($t=p_c$ in this case). The calculations yielded $p_c = 2.11 \pm 0.05$ Å^{-1}.

The $\epsilon(p)$ dependence obtained in this manner is shown in Fig. 2 (curve 1). For comparison, the figure shows the dispersion of the thermal excitations (curve 2), as well as curve 3, the crossing of which should produce an end point of the impurity branch. If p_c does not differ strongly from the characteristic roton momentum p_0, then curve 3 is described by the equation

$$\epsilon_c = \Delta + (p_c - p_0)^2 / 2(m^* + \mu), \qquad (5)$$

where Δ and μ are the roton parameters.

It should be noted that the present result supports to a certain degree Pitaevskiĭ's hypothesis (see[8]), in which the essential factor is apparently not the minimum on the $\epsilon(p)$ curve but the high density of states at large momenta. In addition, the analysis presented here does not exclude the possible existence of a shallow minimum on curve 1 in the region of the roton momentum p_0, but further research is necessary for a final answer to the question of the minimum.

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