The domain structure of vacuum

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We investigate the periodic domain structure of vacuum and the particle spectrum when discrete symmetry is spontaneously broken.

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Inhomogeneous energy-minimizing solutions for vacuum have been considered recently in a number of field-theory models with spontaneously broken symmetry.[1,2] It is shown in[1] that for a scalar (pseudoscalar) field ϕ with Lagrangian

$$L = -\frac{1}{2}\left(\frac{\partial\phi}{\partial x_\mu}\right)^2 - C(\phi^2 - \sigma^2)^2, \qquad C > 0 \tag{1}$$

it is possible to have, besides the homogeneous vacuum state $\phi_h = \pm\sigma$, also an inhomogeneous stationary solution $\phi_u = \sigma\tanh(\mu x/\sqrt{2})$ (where $\mu^2 = 4C\sigma^2$) corresponding to a local energy minimum. Interest in models of this kind is due to the fact that the Lagrangian (1) can be used, in particular, to describe spontaneous CP-violation of vacuum.[3,1]

The formation of a domain wall is attributed in[1] to the fact that the anomalous vacuum mean value ϕ_h in causally-unconnected regions of the universe is independent of the sign in the phase transition to an ordered state (with $\phi \neq 0$). The dimension of the domain is estimated on the basis of the same considerations.

We note in this connection that the equation of motion corresponding to the Lagrangian (1) admits of stationary solutions

$$\phi_0 = \sigma\left(\frac{2k^2}{1+k^2}\right)^{1/2} \mathrm{sn}\{\alpha;k\} \tag{2}$$

($\alpha = \mu(1+k^2)^{-1/2}x$, k is the modulus of the elliptic function, $0 < k^2 < 1$), which describes a periodic domain structure of vacuum with a period

$$D = 2K(k)\mu^{-1}(1+k^2)^{1/2}, \tag{3}$$

where $K(k)$ is a complete elliptic integral of the first kind. In the limiting case $k = 1$, ϕ_0 goes over into ϕ_u.

The absolute minimum of the energy is always reached in homogeneous vacuum $\phi_h^2 = \sigma^2$. Nevertheless, during the course of a relativistic phase transition[4] the phase trajectory of the thermodynamic system of the field ϕ can pass near one of the states (2), which correspond, at k close to unity, to a local energy minimum. The only way out of this state is via fluctuations. We show below that without allowance for the fluctuations the lifetime of a periodic domain structure with $D \gg \mu^{-1}$ can be cosmologically large.

The presence of a periodic domain structure of vacuum leads to nontrivial consequences for the spectrum of the observable particles.

Small deviations ψ from the vacuum solution ϕ_0 satisfy the Lame equation in the Jacobi form[5]

$$\frac{d^2\psi}{d\alpha^2} = \left\{6k^2\,\mathrm{sn}^2\alpha + (1+k^2)\left(\frac{E^2 - q_\perp^2}{\mu^2} - 1\right)\right\}\psi, \tag{4}$$

where E is the energy of the ϕ-field quanta and $q_\perp$ is the momentum of the particle in the plane of the domain walls. Equation (4) is the Schrödinger equation with a nonperiodic potential. Its solutions are the Bloch functions

$$\psi = \frac{H(\alpha+\alpha_1)H(\alpha+\alpha_2)}{\theta^2(\alpha)}e^{iqx}, \tag{5}$$

where $H(\alpha)$ and $\theta(\alpha)$ are the Jacobi eta and theta functions; α_1 and α_2 are parameters that are functions of E and q (see[5]).

The band structure of the spectrum $E(q)$ is determined by transcendental parametric equations in elliptic functions. The particle dispersion law is illustrated in Fig. 1. Here

$$m_1^2(k) = \frac{3}{1+k^2};$$
$$m_2^2(k) = \frac{6k^2}{(1+k^2)(1+k^2-\sqrt{1-k^2+k^4})} - 1 \tag{6}$$

$$\epsilon_1^2(k) = k^2 m_1^2(k);$$
$$\epsilon_0^2(k) = \frac{6k^2}{(1+k^2)(1+k^2+\sqrt{1-k^2+k^4})} - 1. \tag{7}$$

As seen from Fig. 1, in a vacuum with a periodic domain structure there can exist three types of particles.

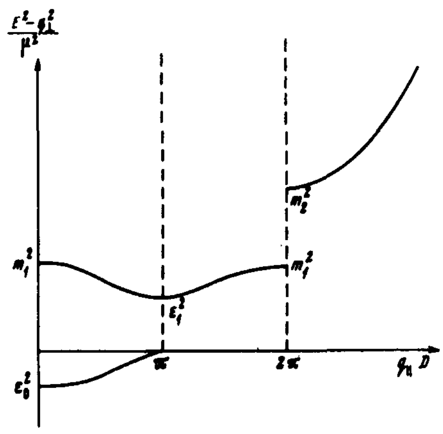

FIG. 1.

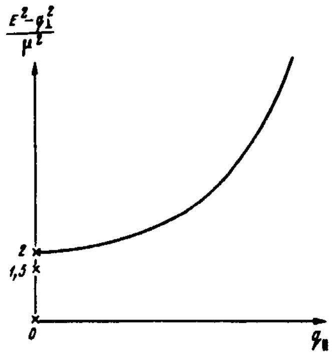

FIG. 2.

At $q_{\parallel}=2\pi/D$ the $E(q)$ curve experiences a discontinuity corresponding to the region of forbidden energies. There are no energy discontinuities on going to higher Brillouin zones.

The particles described by the upper branch of the spectrum have a relativistic dispersion in the region of large momenta $(qD \gg 1)$.

The presence of an excitation branch with $E^2<0$ corresponds to instability of the periodic domain structure for long-wave perturbations $(q_{\parallel}D<\pi)$. At $D \gg \mu^{-1}$ the decay time $\tau=|\epsilon_0|^{-1}$ of the periodic domain structure is given by

$$\tau=\sqrt{\frac{3}{320}}\,\mu^{-1}\exp\frac{D\mu}{\sqrt{2}}\,. \tag{8}$$

Since the characteristic mass μ cannot be much smaller than several GeV,[1] domain structures with $D \geqslant 10^{-11}$ cm can be regarded as relatively stable, since the time τ is cosmologically large in this case. Of course, the true lifetime of the periodic domain structure can be much smaller, owing to its fluctuating transition into the homogeneous state ϕ_h.

In the limiting case of one domain wall $(k=1,\ D=\infty)$, the particle spectrum is shown in Fig. 2. The ordinates represent the limiting values of the masses (these values were obtained in[2]). The particle dispersion laws are then

$$\epsilon_0^2=q_{\perp}^2 \tag{9}$$

$$\epsilon_1^2=m_1^2+q_{\perp}^2 \tag{10}$$

$$\epsilon_2^2=m_2^2+q_{\perp}^2+s^2q_{\parallel}^2 \tag{11}$$

where

$$s=\begin{cases}\frac{1}{2}, & q_{\parallel}\ll\mu\\ 1, & q_{\parallel}\gg\mu\,.\end{cases} \tag{12}$$

The value of the mass $m_2(I)$ coincides with the mass of the excitations over a homogeneous vacuum ϕ_h.

The appearance of a massless branch of the oscillations (9) propagating along the domain wall is due to spontaneous violation of the translational invariance of the vacuum.

We note in conclusion that the onset of an inhomogeneous periodic structure of vacuum, corresponding to an absolute minimum of the energy, is possible when a complex field interacts with gauge fields. This situation, which takes place in type-II superconductors,[6] is presently under investigation by us using concrete gauge-invariant models of weak and electromagnetic interaction.

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