# Elastic backward *pd* scattering at high energies

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The cross section for backward *pd* scattering was calculated in the proton kinetic energy interval from 0.316 to 2.5 GeV within the framework of the pole mechanism, with the relativistic character of particle motion taken into account. The effect of parametrization of the deuteron wave function in relativistic coordinate space is investigated.

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The problem of correctly described the deuteron (or any other nucleus) at relativistic momenta of the intranuclear nucleons, which arises when processes with large momentum transfer are considered, could be solved, generally speaking, by introducing a form factor $F_d(p_1^2, p_2^2)$ that describes the virtual $d \to NN$ decay and depends on the squares of the 4-momenta of the virtual nucleons. A shortcoming of such a description is the 4-

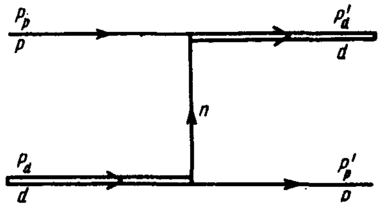


FIG. 1.

dimensional character of the vertex part, which does not permit full use of the available information of the nonrelativistic wave functions to determine the vertex part, thereby imposing a limit on the use of intuitive arguments. Of heuristically greater force is the description of relativistic nuclei with the aid of three-dimensional relativistically-invariant wave functions that have a probabilistic representation and whose representation in the relativistic coordinate space ( $\rho$ space) is determined by the Shapiro transformation.[1] The description of relativistic bound systems with the aid of wave functions in $\rho$ space was discussed in[2]. A more detailed analysis of these functions and their field-theoretical corroboration will be given in subsequent papers. The main task of this communication is to determine the contribution of the relativized pole diagram (Fig. 1) to the cross section of elastic backward $pd$ scattering, describing the deuteron with a three-dimensional wave function, and the effect resulting from parametrization of the deuteron wave function in $\rho$ space. We note that a study of backward $pd$ scattering from various points of view was performed in[3-6].

The cross section of elastic $pd$ scattering, within the framework of the pole mechanism, is given by

$$\frac{d\sigma}{d\Omega} = \frac{3}{4}\left(\frac{E_d E_N}{m(E_d+E_N)}\right)^2 (u-m^2)^2 (u^2(q) + w^2(q))^2, \tag{1}$$

where $E_d$ and $E_N$ are the energies of deuteron and the nucleon in the c.m.s., $u=(p_p-p'_d)^2$, $p_p$ and $p'_d$ are the 4-momenta of the incident proton and emitted deuteron,

$$q = [(m_d-m)^2-u]^{1/2}[(m_d+m)^2-u]^{1/2}/2m_d \tag{2}$$

$m$ and $m_d$ are the masses of the nucleon and deuteron

$$u(q) = \int_0^\infty u(r)\, j_0(qr)\, r\, dr \tag{3a}$$

$$w(q) = \int_0^\infty w(r)\, j_2(qr)\, r\, dr \tag{3b}$$

and $u(r)$ and $w(r)$ are the deuteron wave functions corresponding to the $S$ and $D$ waves.

We shall not dwell on the dynamic questions connected with the variation of the wave functions over small distances. In the derivation of (1), the particle spin was taken into account in nonrelativistic fashion.

In the nonrelativistic case, formula (1) goes over into the expression

$$\frac{d\sigma}{d\Omega} = 3\left(\frac{E_d E_N}{m(E_d+E_N)}\right)^2 (\Delta^2+\kappa^2)^2 (u^2(\Delta) + w^2(\Delta))^2, \tag{4}$$

where

$$\Delta^2 = \left(\mathbf{P}_p - \frac{\mathbf{P}'_d}{2}\right)^2, \quad \kappa^2 = m|\epsilon_d| .$$

We emphasize that the generalization of formula (4) to the relativistic case, which leads to formula (1), consists of replacing the factor $(\Delta^2+\kappa^2)$ and the argument $\Delta$ in the wave function of formula (4) by using different formulas: $\Delta^2+\kappa^2 \to (m^2-u)/2$, and $\Delta \to q$ in accord with formula (2). This approach was used in[6] to describe the energy dependence of the backward $pd$ scattering. In[4], at an energy 1 GeV, the calculation was carried out in accordance with the nonrelativistic formula (4). In[5], the formula (4) was made relativistic by means of the substitution $\Delta^2 \to (m^2-u)/2-\kappa^2$, which does not coincide with formula (2).

The relativistic coordinate space was introduced with the aid of the Shapiro transformation[1]:

$$\Psi(\mathbf{q}) = \int \xi(\mathbf{q},\vec{\rho})\,\Psi(\vec{\rho})\, d^3\rho . \tag{5}$$

The functions $\xi(\mathbf{q},\vec{\rho})$ form a unitary irreducible representation of the Lorentz group and are given by the expressions

$$\xi(\mathbf{q},\vec{\rho}) = \frac{1}{(2\pi)^{3/2}}\left(\frac{E(q)-\mathbf{q}\mathbf{n}}{m}\right)^{-1+i\rho m}, \tag{6}$$

$$\mathbf{n} = \vec{\rho}/\rho .$$

The use of the transformation (5) instead of the Fourier transformation causes the Bessel functions $j_0(qr)$ and $j_2(qr)$ in formulas (3a) and (3b) to be replaced by the functions $S_0(q,\rho)$ and $S_2(q,\rho)$, which are the coefficients of the Legendre-polynomial expansion of $\xi(\mathbf{q},\vec{\rho})$. The functions $S_0(q,\rho)$ and $S_2(q,\rho)$ are given by

$$S_0(q,\rho) = \frac{1}{q\rho}\sin(\rho m\eta) \tag{7a}$$

$$S_2(q,\rho) = \frac{1}{q\rho}\left\{\frac{3m^2}{(\rho m+i)(\rho m+2i)}\sin[(\rho m+2i)\eta] - \frac{3m}{(\rho m+i)q}\cos[(\rho m+i)\eta] - \sin(\rho m\eta)\right\}, \tag{7b}$$

where $\eta$ is the rapidity and

$$\eta = \frac{1}{2}\ln\frac{\sqrt{q^2+m^2}+q}{\sqrt{q^2+m^2}-q} .$$

Since the functions $\xi(\mathbf{q},\vec{\rho})$ at large $\rho(\rho \gg 1/m)$ go over into $(2\pi)^{-3/2}\exp(-i\mathbf{q}\cdot\vec{\rho})$, and the functions $S_0(q,\rho)$ and $S_2(q,\rho)$ go over into spherical Bessel functions, the transformation (5) can be regarded as a method of parametrizing $\Psi(q)$ at relativistic values of $q$; this method leads in continuous fashion to nonrelativistic wave functions at small $q(q<m)$.

The results of the numerical calculations are given in Fig. 2 and in the table. Figure 2 shows the cross section for $pd$ scattering at a proton energy 1 GeV. The solid curves correspond to calculation with ordinary parameterization (formulas (2a) and (3b)), while the dashed curves correspond to calculation with parameterization of the wave function in $\rho$ space (formulas (3a) and (3b) with $j_{0,2}$ replaced by $S_{0,2}$). The function $\Psi(\rho)$ is assumed to coincide with $\Psi(r)$. The two lower curves (solid and dashed) correspond to calculation with the third Moravcsik function,[7] and the two upper ones correspond to the wave function of Humbertson and

| $T_p$, GeV | $q$, GeV/c $\theta = 160°$ | $(d\sigma/d\Omega)^{\text{theor}}$ WF from [7] | | $(d\sigma/d\Omega)^{\text{theor}}$ WF from [8] | | $d\sigma/d\Omega$ experiment | Reference |
|---|---|---|---|---|---|---|---|
| | | $r$ | $\rho$ | $r$ | $\rho$ | | |
| 0.316 | 0.27 | 160 | 178 | 1000 | 1225 | 130 ± 2 | [9] |
| 0.364 | 0.29 | 115 | 124 | 760 | 859 | 150 ± 3 | [9] |
| 0.425 | 0.31 | 80 | 86 | 500 | 572 | 140 ± 10 | [10] |
| 0.470 | 0.33 | 63 | 70 | 380 | 435 | 160 ± 4 | [9] |
| 0.59 | 0.36 | 40 | 45 | 190 | 225 | 140 ± 5 | [9] |
| 1.0 | 0.45 | 17 | 20 | 32 | 44 | 26 ± 2 | [11,12] |
| 1.5 | 0.53 | 8 | 10 | 11.5 | 15 | 3.61 ± 0.15 | [11,13] |
| 2.085 | 0.60 | 3.8 | 5.5 | 10 | 11 | 1 ± 0.13 | [13] |
| 2.485 | 0.64 | 2.3 | 3.7 | 11 | 10 | 0.46 ± 0.1 | [13] |

*Note.* The cross sections are in $\mu b/sr$ throughout; $r$ and $\rho$ denote $r$ space and $\rho$ space, respectively.

Wallace[8] (table). The dash-dot curve is the result of the calculation of Kerman and Kisslinger[4] without allowance for the admixture of the isobars in the deuteron wave function. Since the $S$-wave form factor of the deuteron has a minimum at $q \approx 0.45$ GeV/$c$, the cross section in this range of momentum transfers is determined mainly by the admixture of the $D$ wave in the deuteron wave function. The table lists the values of the cross sections at an angle 160° in the c.m.s. for incident proton energies from 0.316 to 2.485 GeV. We note that the calculation of the cross section at 160° in accord with the nonrelativistic formula (4) with the third Moravcsik function yields a value 8.7 $\mu b/sr$ at 1 GeV. The angular distributions of the cross sections in this energy interval will be published elsewhere.

On the basis of the calculations we can draw the following conclusions: 1) Although the pole mechanism describes the experimental data quite roughly, the pole diagram must be made relativistic in order to take this mechanism accurately into account. 2) The effect of the

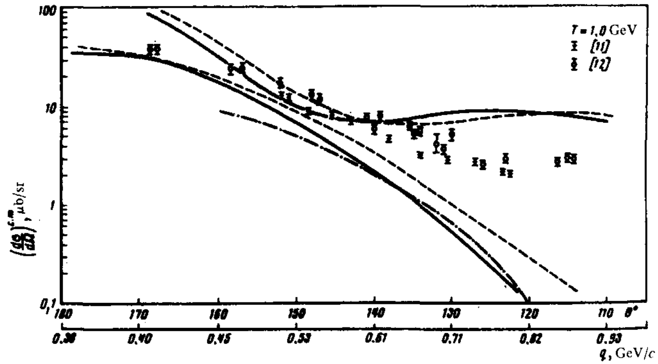


FIG. 2.

parameterization of the wave function in relavistic coordinate space exceeds the experimental errors and is comparable with the difference between the different types of the deuteron wave functions.


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