Structure of developed pion condensate

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A phase transition leading to a change in the structure of the pion condensate in nuclear matter is considered.

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It is known[1,2] that in nuclear matter, at a sufficient density, a phase transition with formation of a pion condensate can occur. In nuclear matter with isotopic symmetry ($N=Z$), the instability sets in simultaneously for all the components of the pion field, and a static electrically-neutral condensate of π^0, π^+, and π^- mesons is produced. At $Z \ll N$, both a π^0 condensate and a condensate of $\pi^+\pi^-$ pairs can be produced.

Pion condensation in the final system (the atomic nucleus) was considered in[4,5]. It was shown that in a sufficiently heavy nucleus the existence of the pion condensate leads to violation of the spherical symmetry. In the present paper it is shown that far enough from the transition point a second phase transition, due to the change in the structure of the condensate can occur in the system. In a nucleus, such a transition would lead to violation of the axial symmetry.

We consider for brevity a neutral condensate. The energy density of the system in the presence of a static field ϕ_0 was obtained in[2]

$$E^\pi(\phi_0) = \sum_{\mathbf{k}} \frac{\tilde{\omega}^2(\mathbf{k})}{2} \phi_{0,\mathbf{k}}\phi_{0,-\mathbf{k}} + \frac{1}{4V}\int \phi_0^4 dV , \qquad (1)$$

where $\tilde{\omega}^2(\mathbf{k}) = 1 + k^2 + \Pi(\mathbf{k},0)$, and $\Pi(\mathbf{k},\omega)$ is the polarization operator of the neutral pions (here and throughout, $\hbar = c = m_\pi = 1$). When the critical density $n_c \approx 0.5 n_0$ is reached (n_0 is the normal nuclear density), states for which $\tilde{\omega}^2 = 0$ appear. With further increase of the density, $\tilde{\omega}^2(k^2) < 0$ at $k_2^2 < k^2 < k_3^2$ and $\tilde{\omega}^2(k_2^2) = \tilde{\omega}^2(k_3^2) = 0$.[2]

Near the transition point, the condensate field is given by

$$\phi_0 = a_0 \sin \mathbf{k}_0 \mathbf{r},$$

$$a_0^2 = 4|\omega_0^2|/3\lambda, \quad \omega_0^2 = \min \tilde{\omega}^2(k^2) = \tilde{\omega}^2(k_0^2), \qquad (2)$$

$\mathbf{k}_0$ is an arbitrarily directed vector, and the condensation energy is $E_0 = -(\omega_0^4/6\lambda)$. Generally speaking, it is necessary to add to the field (2) the higher harmonics $a_1 \sin(3\mathbf{k}_0 \cdot \mathbf{r})$ etc., but near the transition point the amplitude is $a_1 \sim |\omega_0^2|/\tilde{\omega}^2(9k_0^2) \ll 1$ and the harmonics can be neglected.[3] The fact that the minimum energy is given by a one-dimensional structure is due, of course, to pion-pion repulsion.

As shown by numerical calculations,[2] the width of the interval (k_2^2, k_3^2) increases rapidly with increasing density n. The function $\tilde{\omega}^2(k^2)$ changes slowly in the interval $k_2^2 < k^2 < k_3^2$ and increases rapidly with increasing k when $k^2 > k_3^2$, at a density $n \approx n_0$ we have $k_3^2 = 5 - 6k_0^2$, i.e., the region $\tilde{\omega}^2 < 0$ is quite broad. We shall show that under these conditions the one-dimensional structure (2) may turn out to be unstable to transverse oscillations.

We consider the spectrum of above-condensate excitations, determined by the linearized wave equation

$$[\omega^2 - 1 - k^2 - \Pi(\mathbf{k},\omega) - 3\lambda\phi_0^2]\phi' = 0 \qquad (3)$$

$\phi = \phi_0 + \phi'$, $|\phi'| \ll |\phi_0|$; ω and $\mathbf{k}$ must be understood here as the operators $i\partial/\partial t$ and $-i\nabla$. For the field

$$\phi' = [Ae^{i\mathbf{q}\mathbf{r}} + Be^{i(\mathbf{q}+2\mathbf{k}_0)\mathbf{r}} + Ce^{i(\mathbf{q}-2\mathbf{k}_0)\mathbf{r}}]e^{-i\omega t} \qquad (4)$$

and at $\mathbf{q} \perp \mathbf{k}_0$ we obtain the dispersion equation

$$\{P_q\omega^2 - \omega_q^2 - 2|\omega_0^2|\}\{P_+\omega^2 - \omega_+^2 - 2|\omega_0^2|\} - 2\omega_0^4 = 0, \qquad (5)$$

where $\omega_q^2 = \tilde{\omega}^2(\mathbf{q})$, $\omega_+^2 = \tilde{\omega}^2(\mathbf{q}+2\mathbf{k}_0)$, and $P = 1 - \partial\pi/\partial\omega^2$ (since we are interested in the region where ω^2 reverses sign, we have expanded the linearized operator in powers of ω^2). It is easily seen that at

$$\epsilon = \left\{\frac{1}{2} - \left(1 - \frac{\omega_q^2}{2\omega_0^2}\right)\left(1 - \frac{\omega_+^2}{2\omega_0^2}\right)\right\} > 0 \qquad (6)$$

the dispersion equation (5) has solutions with $\omega^2 < 0$, thus pointing to the instability of the one-dimensional structure (2). Thus, at the density n_c' at which ϵ reverses sign, a second phase transition occurs and is due to the change of the structure of the condensate. The form of the equilibrium structure near the point of the second transition (at $\epsilon \ll 1$) can be easily obtained by assuming, as before, that $|\omega_0^2|/\tilde{\omega}^2(9k_0^2) \ll 1$ (the latter condition greatly simplifies the calculations)

$$\phi = a \sin \mathbf{k}\mathbf{r}(1 + \xi \cos 2\mathbf{q}\mathbf{r}) + b \sin \mathbf{q}\mathbf{r}(1 + \zeta c \cos 2\mathbf{k}\mathbf{r}). \qquad (7)$$

We shall not present the rather cumbersome exact values of the coefficients in (7), and indicate only that

$k_0^2 - k^2 \sim \epsilon k_0^2$, $a_0^2 - a^2 \sim b^2 \sim \epsilon a_0^2$, $\zeta \sim 1$, and $\xi \sim \epsilon$. The value of q is determined from the condition $\partial\epsilon/\partial q = 0$. The correction to the condensation energy is then $\Delta E = E - E_0 \sim \epsilon^2 E_0$. We note that owing to averaging over the volume the terms of order $a_0 b^3$ vanish, and no first-order transition can take place. Thus, in the mode considered here, the equilibrium state is the two-dimensional lattice (7). It can be shown that a three-dimensional structure of the type (7) is energywise not favored.

As noted in[4,5] pion condensation leads to violation of the spherical symmetry of the nucleus, both because of the change in its shape and because of the modulation of the nucleon density. The formation of the two-dimensional structure (7) would lead, for the same reasons, to violation of the axial symmetry of the nucleus.

We note in conclusion that the possibility of a phase transition with change of the structure should be taken into account in calculations of the influence of the pion condensate on the properties of a neutron star.[6,7]

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