Multiwave decay interactions

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The stabilization of parametric instability of coherent plasma oscillations is investigated in the case when the energy transfer from the instability region into the dissipation region is effected by dynamic decay processes.

A number of papers[1-4] have been devoted to stabilization of parametric instability via spectral transfer of plasma oscillations from the instability region to the region of collisional dissipation, under the assumption that the main plasmon-transfer process is induced scattering by ions. It is known that in a nonisothermal plasma the processes with lower thresholds are decay interactions of waves with participation of weakly-damped ion-sound oscillations. In contrast to the induced scattering of waves by particles, when decays in a coherent pumping field are considered it is necessary to take into account the phase relations between the interacting waves, and this greatly complicates the problem. In this paper we investigate the possibility of stabilizing parametric instability of coherent plasma oscillations by dynamic decay transfer of their energy into the dissipation region.

We consider a thin plasma layer placed in a uniform high-frequency field $\mathbf{E}=\mathbf{E}_0\exp(i\omega t)$, with an electric-field vector parallel to the boundary of the layer, and with a frequency close to the plasma frequency, $\omega\approx\omega_{pe}$. If the amplitude of the external field is higher than that of the threshold field, then the decay instability will excite in the plasma high-frequency Langmuir oscillations of frequency ω_0 and wave vector $\mathbf{k}_0$, and a low-frequency ion-sound wave $(\Omega_1,\vec{\kappa}_1)$ propagating in the direction of $\mathbf{E}_0$, with the following synchronism conditions satisfied:

$$\omega=\omega_0+\Omega_1,\quad \mathbf{k}_0=-\vec{\kappa}_1. \tag{1}$$

The growing high-frequency plasma wave can decay in turn into an opposing Langmuir wave $(\omega_1,\mathbf{k}_1)$ and a new low-frequency wave $(\Omega_2,\vec{\kappa}_2)$

$$\omega_0=\omega_1+\Omega_2,\quad \mathbf{k}_0=\mathbf{k}_1+\kappa_2. \tag{2}$$

The succeeding decays and coalescence of the high-frequency and low-frequency waves can give rise to waves with frequencies $\omega_{\pm n}=\omega_0\pm n\Omega_2$ (n is an integer) and corresponding wave vectors $\mathbf{k}_{\pm n}=\mathbf{k}_0\pm n\vec{\kappa}_2$. These high-frequency waves are also natural oscillations of the plasma[1)] if the condition $k_0r_d\gg(2n/3)\sqrt{m/M}$ is satisfied, i.e., under this condition it can be assumed that the "decay" transfer of plasmon is effected via one low-frequency wave.[2)]

The initial system of equations for the complex amplitudes of the interacting waves will be represented in the form

$$\frac{da_0}{d\tau}=-a_0+Fb_1^*+a_1b_2^*-a_{-1}b_2,$$

$$\frac{da_n}{d\tau}=-a_n+a_{n+1}b_2^*-a_{n-1}b_2,$$

$$\frac{db_1}{d\tau}=-\delta_1b_1+Fa_0^*,$$

$$\frac{db_2}{d\tau}=-\delta_2b_2+\sum_n a_n^*a_{n+1}. \tag{3}$$

Here $a_n=u_n\beta/\gamma$, $b_n=v_n\beta/\gamma$, $F=u_p\beta/\gamma$, where u_n, v_n, and u_p are respectively the amplitudes of the high-frequency, low-frequency and pump waves, $\tau=\gamma_0t$ is the dimensionless time, $\delta_n=\Gamma_n/\gamma_0$, while $\gamma_n=\gamma_0$, and Γ_n are the damping decrements of the high-frequency and low-frequency waves, and $\beta_n=\beta$ are the coefficients of the decay interaction and are assumed to be independent of the number n.

The system (3) differs from the conservative system considered in[5] in the presence of negative absorption at the zeroth harmonic and in the presence of dissipation in all the remaining waves. This system can also be used as a model in other problems involving stabilization of coherent oscillations via "decay" renormalization of the spectrum.

An analysis of the solutions (3) performed with the aid of the generating-function method shows that the simplified system (3) has no stationary solutions,[3)] i.e., the total energy of the high-frequency waves increases without limit in time, $\sum_n|a_n|^2\to\infty$. To assess the possibility of stabilizing a parametric instability it is therefore necessary to take into account additional factors, such as the dependence of the interaction coefficients β_n on the number n of the harmonic, the violation of the synchronism of the interacting waves, the dependence of the damping decrements of the waves on the amplitudes, etc. Which of these factors is decisive depends on the particular physical system with which the model equations are set in correspondence. In the case considered by us, that of decay instability of Langmuir oscillations, the most important is allowance for the interaction of the low-frequency waves, for by virtue of the kinematic conditions (1) and (2) the ion-sound wave b_2 is the second harmonic of the wave b_1 ($\kappa_2=2\kappa_1$ and $\Omega_2=2\Omega_1$). Taking this interaction into account and introducing the generating function $\phi=\sum_na_n\exp(in\phi)$, we obtain the system of equations

$$\frac{d\Phi}{d\tau}=-\Phi+Fb_1^*+b_2^*\Phi e^{-i\phi}-b_2\Phi e^{i\phi}$$

$$\frac{db_1}{d\tau}=-\delta_1b_1+F\langle\Phi^*\rangle-\mu_1b_1^*b_2, \tag{4}$$

$$\frac{db_2}{d\tau}=-\delta_2b_2+\langle\Phi\Phi^*e^{-i\phi}\rangle+\mu_2b_1^2$$

The angle brackets ⟨ ⟩ denote here averaging over ϕ, while μ_1 and μ_2 are the nonlinear-interaction coefficients.

Analytic expressions for the stationary states of the system (3) can be obtained only in certain limiting cases:

1. Under the condition $\mu_1 F^2/2\delta_1^2 \ll 1$ we have

$$|b_1|^2 = \frac{\delta_2}{\mu_2}\,\frac{\sqrt{F^4-\delta_1^2}}{2\delta_1}, \quad |b_2|^2 = \frac{F^4-\delta_1^2}{4\delta_1^2}, \tag{5}$$

i.e., the spectrum is symmetrical about the zeroth harmonic $n=0$ and falls off rapidly with increasing $|n|$. At a pump amplitude close to the threshold value ($F^2-\delta_1 \ll \delta_1$) we can obtain the following expression for the total energy of the high-frequency waves

$$\frac{W_\Sigma}{NT_e} = 16\pi\frac{m}{M}\left(\frac{E_{\text{pump}}-E_{\text{thr}}}{E_{\text{thr}}}\right)^{1/2} \tag{6}$$

2. At a large excess over threshold ($F^2 \gg \delta_1$) and under the additional conditions $\sqrt{\mu_1}F/\delta_1 \gg 1$ and $F^2 \gg \mu_1$ we obtain

$$|a_n|^2 = \frac{\delta_2\sqrt{\mu_1}F}{2\sqrt{2}\mu_2}\left(1-\frac{\sqrt{2\mu_1}}{F}\right)^{|n|},$$

$$\frac{W_\Sigma}{NT_e} = \sqrt{\frac{\pi}{2}}\,\frac{\omega_{pe}}{\gamma_l}\sqrt{\frac{m}{M}}\,\frac{E^2_{\text{pump}}}{2\pi NT_e} \approx 8\pi\frac{m}{M}\,\frac{E^2_{\text{pump}}}{E^2_{\text{thr}}}. \tag{7}$$

A numerical investigation of the temporal evolution of the solutions of the system (4) has shown that these solutions approach the stationary states obtained above in the course of time (see the figure).

The derived relations can be used to estimate the effectiveness of anomalous energy dissipation of an electromagnetic wave in parametric decay instability. It follows from them, in particular, that at pump amplitudes appreciably higher than threshold we get saturation of the effective collision frequency

$$\nu_{eff} = \frac{8\sqrt{\pi}}{\sqrt{2}}\,\omega_{pe}\sqrt{\frac{m}{M}}.$$

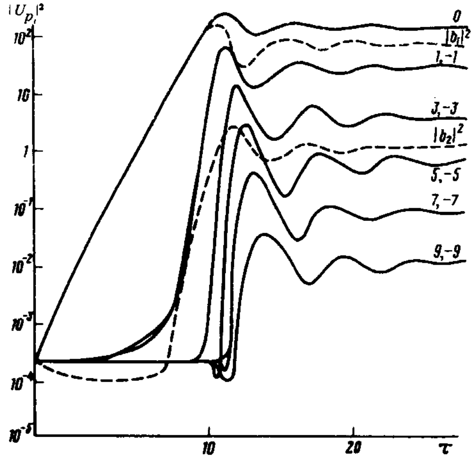

FIG. 1.

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1)We disregard induced plasma oscillations produced in interaction with the low-frequency wave $(\Omega_1, \vec{\kappa}_1)$.

2)We propose to explain the conditions for the applicability of this approximation in greater datail by a numerical investigation of the system of equation that describes one-dimensional dynamic Langmuir turbulence.[6]

3)Stationary states of this kind appear if small noise sources are introduced into Eq. (3).

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