Magnetization dynamics in the dipole critical region

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The Wilson method is used to investigate the role of dipole forces in the critical dynamics of the magnetization of a ferromagnet.

The Wilson renormalization-group approach to critical phenomena[1] was recently generalized to an investigation of the Ginzburg-Landau model with exchange interaction.[2] In magnets, however, there is always also a dipole interaction. In the region of temperatures T which is most important for phase transitions, i.e., barely above the critical T_c, when the difference $T-T_c$ is comparable with the energy of the dipole interaction, it can become essential in the critical behavior of the system. The static properties of the critical behavior in this dipole region were investigated by the ϵ-expansion method by Fisher and Aharony,[3] and the dynamic properties will be analyzed below.

A characteristic feature of the dipole forces is their long-range action and divergence, which makes it necessary to introduce demagnetizing factor. Of no little importance in the use of the ϵ expansion is also the fact that from the form of the isotropic dipole Hamiltonian it follows that the number of components of the order parameter n and the dimensionality d of the system are equal. Finally, the dipole interaction leads to nonconservation of the order parameter, so that we can assume constancy of the damping decrement Γ in the Ginzburg-Landau equation that simulates the low-frequency dynamics

$$\frac{\partial \sigma_{\mathbf{k}}^{\alpha}}{\partial t} = -\Gamma \frac{\delta \bar{\mathcal{H}}}{\delta \sigma_{-\mathbf{k}}^{\alpha}} + \eta_{\mathbf{k}}^{\alpha} \tag{1}$$

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Here σ_k^α is the Fourier transform of the order-parameter component ($\alpha = 1, 2, \ldots, n$), $\mathbf{k}$ is the wave vector and has an upper bound, and η_k^α is the component of the random force that characterizes the fluctuations of the infinite thermal reservoir. In this model we neglect effects connected with conservation of the energy density. For system with a specific heat that is constant at $T = T_c$, such as ours $[\alpha_s = -(1/34)\epsilon < 0]$, [3] this will not affect the results concerning the order parameter. [2] Substituting in (1) the modified Hamiltonian $\bar{H}$ of[3] with allowance for the Zeeman energy in a weak magnetic field $\mathbf{h}_k(t)$ we obtain the system of differential equations

$$\frac{\partial \sigma_{\mathbf{k}}^a}{\partial t} + \Gamma(r_0 + k^2)\sigma_{\mathbf{k}}^a + \Gamma \sum_{a'} (g_0 - h_0 k^2)\frac{k^a k^{a'}}{k^2}\sigma_{\mathbf{k}}^{a'} = \Gamma h_{\mathbf{k}}^a + \eta_{\mathbf{k}}^a - 4u_0 \Gamma \sum_{a'} \iint d^d k_1 d^d k_2 \sigma_{\mathbf{k}_1}^{a'} \sigma_{\mathbf{k}_2}^{a'} \sigma_{\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2}^a . \tag{2}$$

Here $r_0 \sim T$; g_0 and h_0 are parameters proportional to the dipole-interaction constants, and u_0 is the parameter of the nonlinear coupling of the fluctuations. The requirement that the solutions of (2) lead in the equilibrium state to the static binary correlators of[3] imposes on the mean values of η_k^α the Gaussian conditions $\langle \eta_k^\alpha(t) \rangle = 0$, $\langle \eta_k^\alpha \eta_{k'}^{\alpha'} \rangle = 2\Gamma\delta(\mathbf{k} + \mathbf{k}')\delta(t - t')\delta_{\alpha\alpha'}$.

Changing over to continuous values of the system dimensionality d and taking into account the fact that according to Wilson, an important role is played in the vicinity of the phase transition by a fixed value of the Hamiltonian where in the presence of dipole forces and at $\epsilon = 4 - d > 0$ we have $8\pi^2 u_0 \approx \epsilon/34$, [3] we obtain from the system (2), by iterating with respect to u_0, an expression for the dynamic magnetic susceptibility

$$\chi_{\alpha\beta}^{-1}(\mathbf{k}, \omega) = G_{\alpha\beta}^{-1}(\mathbf{k}, \omega) + \Sigma_{\alpha\beta}(\mathbf{k}, \omega) . \tag{3}$$

The self-energy part $\sum_{\alpha\beta}$ corresponds to graphic expansion in powers of u_0, and the nonrenormalized correlator is

$$G_{\alpha\beta}(\mathbf{k}, \omega) = \frac{\delta_{\alpha\beta} - k^\alpha k^\beta / k^2}{r_0 + k^2 - i\omega/\Gamma} + \frac{k^\alpha k^\beta / k^2}{g_0 + r_0 + (1 - h_0)k^2 - i\omega/\Gamma}, \quad k \neq 0;$$
$$G_{\alpha\beta}(0, \omega) = \delta_{\alpha\beta}(r_0 + g_0 D_\alpha - i\omega/\Gamma)^{-1} \tag{4}$$

The last formula, where D_α is the principal value of the demagnetizing-coefficient tensor, [3] was written out for an ellipsoidal sample.

To analyze the homogeneous susceptibility $\chi_{\alpha\beta}(0, \omega)$ at $T = T_c$ and at low values of the frequency ω, it is convenient to use a renormalized expansion, where $r_0(T_c)$ is substracted from $r_0(T)$ in (3) and in (4). It is then necessary to subtract $\sum_{\alpha\beta}(0, 0)$ from the self-energy part. Without going into details of the graphic expansion, we note that the first nonvanishing correction to the nonrenormalized correlator in (3) appears in second order in u_0. At small ϵ we confine ourselves to this accuracy. The transverse and longitudinal parts of $G_{\alpha\beta}$ yield in the logarithmic approximation corrections proportional to $\ln\omega$ $\ln g_0$, respectively. Asymptotically as $\omega \to 0$ we can neglect the contribution of the longitudinal fluctuations to $\sum_{\alpha\beta}$. Calculations, as well as the relation $\omega(1 - \lambda \ln\omega) \approx \omega^{1-\lambda}$ which holds at $\lambda \ll 1$, lead to

$$\chi_{\alpha\beta}^{-1}(0, \omega) \approx \delta_{\alpha\beta}(g_0 D_\alpha - i\Gamma^{-1}\omega^{1-\lambda}), \qquad \lambda = \frac{90}{(34)^2}\epsilon^2 \ln\frac{4}{3} . \tag{5}$$

The term $\chi_{\alpha\beta}^{-1}$ which depends on ω can be represented by the form traditional of dynamic similarity[1)] $k^{2-\eta} f(\omega/\Gamma k^z)$ taken at $k = 0$ (the static exponent is[3] $\eta = [80\epsilon^2/3(34)^2]$, and the dynamic critical exponent is

$$z = 2 + c\eta, \qquad c = \frac{27}{4}\ln\frac{4}{3} - 1 . \tag{6}$$

The fact that (5) and (6) differ from the results of[2] indicates that the critical dynamics possessed by models with short-range exchange forces has given way to long-range dipole forces. In analogy with the static case, this change, which is due to the presence of the parameter g_0 in the Hamiltonian, can be characterized by the dynamic crossover exponent $\bar{\psi}$, introduced by the relation

$$\chi_{T=T_c}(\omega, g_0) \sim \omega^{-(1-\lambda)}\Phi(g_0/\omega^{\bar{\psi}}) = g_0^{-(1-\lambda)/\bar{\psi}}\tilde{\Phi}(g_0/\omega^{\bar{\psi}}). \tag{7}$$

It follows from (5) that in our case $\bar{\psi} \approx 1 - \lambda$.

Let us consider the temperature ($T > T_c$) dependence of the relaxation time of the order parameter T_σ. Calculating the imaginary part of $\chi_{\alpha\beta}^{-1}(0, \omega)$ as $\omega \to 0$ and substituting $r_0(T) - r_0(T_c) \approx B\tau^\gamma$, where $\tau = (T - T_c)/T_c$ and γ is the exponent of the static susceptibility, [3] we obtain

$$T_\sigma \sim \Gamma^{-1}(B\tau^\gamma + g_0 D_\alpha)^{-1}\tau^{-\Delta} \sim \tau^{-\Delta_\alpha}, \qquad \Delta = \frac{1}{2}(c + 1)\eta . \tag{8}$$

From a comparison of (8) with the results of[4] it follows that at $D_\alpha \neq 0$ the critical exponent of the order-parameter relaxation time Δ_α ($= \Delta$) is smaller by almost unity than in the exchange region. This fact is probably easiest to observe experimentally. We emphasize that the "dipole" behavior takes place in a narrow ($\tau \sim 10^{-2} - 10^{-3}$) region of temperatures near T_c (see[3] for details). If $D_\alpha = 0$, then $\Delta_\alpha = \Delta + \gamma$ and differs only little from the exchange value, and in this case the dynamic similarity relation $\Delta_\alpha = \nu z$ is satisfied (ν is the critical exponent of the correlation radius[3]).

Our results, which correspond at $\epsilon = 1$ to the usual three-dimensional situation, were obtained without using the dynamic-similarity formulas, and refine the latter somewhat. Mention should be made of recent results ($z \approx 1$, $\Delta_\alpha \approx 2/3$) of a different approach to the considered problem. [5] It is not excluded that the disparity between these results and (6) and (8) is due to the neglect of demagnetization effects in[5].

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1)The validity of this statement can be demonstrated also for finite k and ω.

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