Optimal plasma compression in Z and Θ pinches

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The feasibility of adiabatic compression of a uniform plasma, not accompanied by the development of shock waves, is demonstrated. The laws governing the optimal growth of the external magnetic field (Θ pinch) and of the longitudinal current (Z pinch) are obtained.

To obtain an ultradense plasma by laser heating of a target or in a Z or Θ pinch it is necessary to shape specially the time waveform of the compression pulse. The optimal regime is then reached under conditions when no shock waves (SW) develop in the medium and the propagation velocity of the thermal waves (TW) is low, i.e., there is no preliminary heating that hinders the compression.

The problem of optimizing the laser experiment was studied earlier in[1] for compression of substances with equations of state $p \sim P^{\gamma}$, but only the cases $\gamma = 5/3$ and $\gamma = 3$ were investigated. In this paper we consider the adiabatic compression of an infinite cylindrical plasma column of finite mass, for the purpose of determining the optimal laws governing the growth of the longitudinal current $I(t)$ and the external magnetic field $B_e(t)$ in Z and Θ pinches. If the internal magnetic field B_i of the plasma is not equal to zero, is parallel to the column axis, and the freezing-in condition is satisfied, then by using the known two-dimensional adiabat[2] we can reduce the transverse compression to hydrodynamic adiabatic flow, but with an adiabatic exponent $\gamma = 2$.[3] It suffices here to introduce, in place of the plasma pressure $p_\perp$ "transverse" to the field, a pressure $p = p_\perp + B_i^2/8\pi$. Thus, the case $\gamma = 2$ is of interest for plasma pinches. It should also be noted that unlike the previously investigated power-law changes of the plasma-column parameters,[4] when the plasma velocity is proportional to the distance from the axis, we obtain the laws governing the optimal compression of a uniform plasma initially at rest.

We consider thus nondissipative adiabatic compression of a plasma column. At arbitrary γ it is described by the equations (we assume cylindrical symmetry through out and designate by "0" the initial values of the quantities):

$$\dot v \equiv \left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial r}\right)v = -\frac{2}{\gamma-1}\, c\,\frac{\partial c}{\partial r}; \quad \frac{2}{\gamma-1}\,\frac{\dot c}{c} = -\frac{1}{r}\,\frac{\partial}{\partial r} r v, \qquad (1)$$

where r is the Euler coordinate, while $v = \dot r$, $c = \sqrt{\gamma P/\rho}$, and p are the speed of sound, the density, and the pressure. Assume that the compression begins at the instant $t = 0$ and the law of motion of the outer boundary is $R(t)$, then the supplementary conditions are

$$v(r,\,0) = 0; \quad c(r,\,0) = c_0; \quad v(R(t),\,t) = \dot R(t); \quad R(0) = R_0 \qquad (2)$$

If $R(t)$ is chosen such that the SW develops at an instant of time $t_1 = R_0/c_0$ such that $R(t_1) = 0$ then, first, the given $R(t)$ is optimal in the sense indicated above, and second the flow, which differs from the initial one in the region $R_f(t) \le r \le R(t)$, where $R_f(t) = c_0(t_1 - t)$ is the law of motion of the sound-wave front, is self-similar.[1] It is therefore sufficient to have solutions of (1) in the form $v(r,\,t) = v(\xi)$ and $c(r,\,t) = c(\xi)$, where $\xi = r/(t_1 - t)$, subject to the supplementary conditions

$$v(R_f(t),\,t) = 0; \quad c(R_f(t),\,t) = c_0. \qquad (3)$$

It is convenient to write down the complete system of equations, describing the problem, for the functions $z(t)$, $y(t)$ and $\tilde\xi(t)$ defined by the relations $\tilde\xi(t) = R(t)(t_1 - t)^{-1}$, $v(\tilde\xi(t)) = -\tilde\xi(t)y(t)$, and $c(\tilde\xi(t)) = \tilde\xi(t)(1-y)\sqrt{1+z(t)}$. Using conditions (3) and the relation

$$(\tilde\xi)^{\frac{2\gamma}{\gamma-1}}(1-y)^{\frac{\gamma+1}{\gamma-1}}(1+z)^{\frac{1}{\gamma-1}} = \frac{A}{(t_1-t)^2}; \quad A = t_1^2(c_0)^{\frac{2\gamma}{\gamma-1}},$$

which is the consequence of the conservation of the total compressed mass, we obtain ($0 \le t \le t_1$):

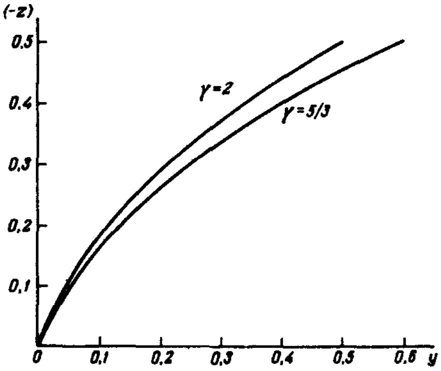

FIG. 1. Optimal trajectories $z(y)$ of the system (4).

$$\frac{d}{dt}y=-2\frac{y(1-y)}{t_1-t}\,\frac{z+1/2}{z},\quad y(0)=0,$$

$$\frac{d}{dt}z=-2\frac{1+y}{t_1-t}\left[z+\frac{y+1}{2}\,\frac{y}{1+y}\right]\frac{z+1}{z},\quad z(0)=0. \tag{4}$$

As shown by the analysis of the trajectories $z(y)$ of the system (4), there exists a unique trajectory (see Fig. 1), which starts at the point ($z=0$, $y=0$) at $t=0$ and ends up at the point ($z=-1/2$, $y=1/\gamma$) at $t=t_1$; this trajectory ensures that $R(t_1)=0$ and is by the same token optimal.

To determine the laws governing the growth of the external field $B_e(t)$ and of the current $I(t)$ in the case of skin-layer pinches, it suffices to use the equilibrium condition for the pressures on the outer boundary, $B_e^2=8\pi p$, and for the connection between the longitudinal current and the field produced by it, $I\sim B_eR$. The optimal $I(t)$ and $B_e(t)$ obtained in this manner with the aid of numerical integration of Eqs. (4) for $\gamma=5/3$ ($B_{i0}=0$) and $\gamma=2$($B_{i0}\neq 0$) are shown in Fig. 3, which indicates also the laws governing the change of the pinch radius $R(t)$ and the input power

$$\frac{W}{W_0}=\left[\left(\frac{c}{c_0}\right)^{\frac{2\gamma}{\gamma-1}}\frac{v}{c_0}\,\frac{R}{R_0}\right]_{r=R(t)}.$$

Let us examine the influence of the finite thermal conductivity of the plasma. As is well known, the coefficient of the thermal diffusivity is

$$\chi\sim\frac{T^{5/2}}{n}\left(1+\mathrm{const}\frac{B_i^2T^3}{n^2}\right)^{-1},$$

where T and n are the temperature and particle-number density in the plasma. If $B_i\equiv 0$, recognizing that $p=nT\sim n^\gamma$, this yields $\chi\sim(n)^{(5\gamma-7)/2}$. Consequently, at $\gamma>7/5$, the quantity χ, which characterizes the rate of propagation of the TW, increases with density. Therefore allowance for the thermal conductivity can greatly alter the optimal compression regime. If $B_i\neq 0$, then, as shown above, the optimal variant corresponds to $p=nT\sim n^2$ and $B_i\sim n$, therefore $\chi\sim 1/n^{3/2}$ as $n\to\infty$, i.e., in a sufficiently strong initial internal magnetic field the TW may be "frozen," and consequently allowance for the thermal conductivity does not change significantly the obtained optimal compression law. For example, as shown by estimates, at an initial deuterium plasma density $n_0=10^{15}$ cm^{-3} and $P_{i0}/n_0=27$ eV, an initial field $B_{i0}=10^2$ G suffices for the compression wave to negotiate each centimeter of the unperturbed medium at a speed larger by one order of magnitude than the TW.

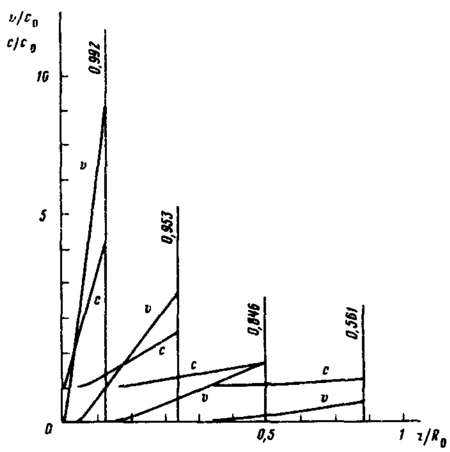

FIG. 2. Velocity profiles along the radius of the pinch at fixed instants of time, $\gamma=5/3$ (the parameter of the curves is t/t_1).

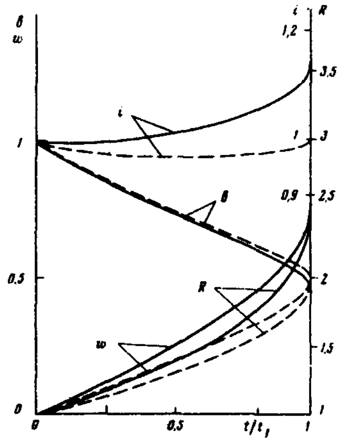

FIG. 3. Time variation of the pinch radius $R(t)$, of the external field $B_e(t)$, of the current $I(t)$, and of the power $W(t)$ in the optimal regime (solid curves —$\gamma=5/3$, dashed—$\gamma=2$). The notation is: $R=[R(t)/R_0](1-t/t_1)^{-1/\gamma}$; $w=[W(t)/W_0](1-t/t_1)^{(3-2)/\gamma}$; $i=[J(t)/I_0](1-t/t_1)^{(\gamma-1)/\gamma}$; $b=[B_e(t)/B_{e0}](1-t/t_1)$.

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