Electromagnetic corrections to the production of narrow resonances in colliding e^+e^- beams

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Formulas that are exact in the parameter $(4\alpha/\pi)\ln(W/m_e)\ln(W/\Delta E)$ were obtained for the radiative corrections to the cross sections for the production of narrow resonances in colliding e^+e^- beam experiments. Phenomena connected with the interference of the resonance with the nonresonant background are discussed.

1. Recent experiments with colliding e^+e^- beams have revealed the narrow resonances $\psi(3105)$ and $\psi'(3695)$.[1,2]

An important role is played in the production of narrow resonance by radiative corrections (RC), a fact first noted in[3] (see also[4]). Allowance for the emission of soft quanta greatly alters the shape of the resonance curve, for if the total beam energy W exceeds the resonance mass M, then γ-ray emission that returns the electrons to resonance becomes more favored. In addition, the contribution of the virtual quanta leads to a strong suppression of the cross section. The parameter that determines the RC, as is well known, is equal to $(4\alpha/\pi)\ln(W/m_e)\ln(W/\Delta E)$. The specific feature of narrow resonances is that ΔE is quite small even at poor accuracy of the measurement of the final-particle energies. The point is that the emission of quanta of high frequency is cut off by a rapid decrease of the reresonance cross section. Therefore the value of ΔE is determined by the larger of three quantities, namely the resonance width Γ, the spread σ of the total energy of the beams (a Gaussian distribution is assumed), and $W-M$. We present below formulas that describe the RC in the case of narrow resonances (under the assumption that the resonance spin is $J=1$). They differ from the expressions obtained in[3,4] in that they are accurate in the large parameter $(4\alpha/\pi)\ln(W/m_e)\ln(W/\Delta E)$, and also contain an interference with the nonresonant background.

The formula that takes into account the interaction of the initial electrons with virtual photons and the emission by them of an arbitrary number of soft photons with a total energy $\omega<\omega_0\ll W$ is of the form

$$\sigma(W) = \int_0^{\omega_0} d\omega\, \sigma_0(W-\omega)\frac{d}{d\omega}\left(\frac{2\omega}{W}\right)^{\beta}(1+{}^3/_4\,\beta), \tag{1}$$

where $\beta=(4\alpha/\pi)[\ln(W/m_e)-1/2]$ and $\sigma_0(W)$ is the cross section of the process without the RC. In the derivation of (1) we have neglected terms of order α and β^2 as well as diagrams with vacuum polarization. The polarization of vacuum by a photon going into resonance is included in the width $\Gamma_{e^+e^-}$ of its decay into e^+e^- (which is regarded as a quantity that is inclusive with respect to the γ rays). In the remaining cases the contributions of these diagrams are numerically much smaller than β. A similar smallness obtains also for diagrams with e^+e^- pair production at $\omega_0>2m_e$. Thus, the accuracy of formula (1) is apparently on the order of one per cent. The RC connected with the final particles depend on the experimental conditions and are small if the resolution with respect to their energies is poor.

2. Assuming that the amplitude of the process without the RC consists of a contribution of a Breit-Wigner type and a one-photon background, we obtain from (1) an expression for the total hadron production cross section

$$\sigma^h = \frac{4\pi}{M^2}\left\{\left(1+\frac{3}{4}\beta\right)\frac{3\Gamma_{e^+e^-}}{M}\left(\frac{\Gamma_h}{\Gamma}\right)\mathrm{Im}f - \frac{2\alpha\sqrt{R\Gamma_{e^+e^-}\Gamma_h}}{M}\lambda\left(1+\frac{11}{12}\beta\right)\mathrm{Re}f + \frac{\alpha^2 R}{3}\left(1+\frac{13}{12}\beta\right)\right\}, \qquad f=\left(\frac{\frac{M}{2}}{-W+M-\frac{i\Gamma}{2}}\right)^{1-\beta}. \tag{2}$$

Here Γ_h is the hadron width of the resonance, and $R=\sigma^h/\sigma^{\mu^+\mu^-}$ off resonance. The parameter $\lambda(|\lambda|\leqslant 1)$ characterizes the proximity of the properties of the final states in the decay of the resonance in the one-photon channel.

The total cross section of the process e^+e^- near resonance is described by formula (2) in which (assuming μ-e universality) Γ_h should be replaced by $\Gamma_{e^+e^-}$ and we must put $R=1$ and $\lambda=1$. The angular distribution of the final muons is of the usual form $\sim(1+\cos^2\theta)$. We emphasize that for narrow resonances the asymmetry in the angular distribution of the muons, resulting from the RC, does not contain large logarithms, in contrast to the nonresonant case.

The cross section for elastic e^+e^- scattering is described by the formula

$$\frac{d\sigma^{e^+e^-}}{d\Omega} = \frac{1}{M^2}\left\{\frac{9}{4}\frac{\Gamma^2_{e^+e^-}}{\Gamma M}(1+\cos^2\theta)\,\mathrm{Im}f - \frac{3\alpha}{2}\frac{\Gamma_{e^+e^-}}{M}\left[(1+\cos^2\theta) - \frac{(1+\cos\theta)^2}{(1-\cos\theta)}\right]\mathrm{Re}f + \frac{\alpha^2}{4}\frac{(3+\cos^2\theta)^2}{(1-\cos\theta)^2}\right\}, \tag{3}$$

where the terms proportional to β were omitted. For comparison with the experimental data, formulas (2) and (3) must be averaged over the Gaussian distribution

$$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(W-W_0)^2}{2\sigma^2}\right)$$

We then obtain formulas that differ from (2) and (3) by the substitution

$$f\to\tilde f = e^{-x^2/4}\, e^{i\pi/2\,(1-\beta)}\, D_{\beta-1}(-ix)\left(\frac{2\sigma}{M}\right)^{\beta-1}$$

$$x = \frac{W_0 - M + i\Gamma/2}{\sigma}, \tag{4}$$

where $D_p(z)$ is the parabolic-cylinder function.[5]

The behavior of the cross sections at $W-M\gg\Gamma$, σ does not depend on σ and is given for σ^h by the formula

$$\sigma^h\Big|_{(W-M)>>\Gamma,\,\sigma} = \frac{6\pi^2}{M^2}\frac{\Gamma_h}{\Gamma}\frac{\Gamma_{e^+e^-}}{W-M}\beta\left(\frac{W-M}{M/2}\right)^{\beta}\left[\left(1+\frac{3}{4}\beta\right) + \frac{2\lambda\alpha\sqrt{R}}{3\pi\beta}\sqrt{\frac{\Gamma^2}{\Gamma_h\Gamma_{e^+e^-}}}\left(1+\frac{11}{12}\beta\right)\right] + \frac{4\pi}{3}\alpha^2\frac{R}{M^2}\left(1+\frac{13}{12}\beta\right). \tag{5}$$

Comparing the obtained formulas with the experimental data, we can determine in rather simple fashion the quantities $(\Gamma_{e^+e^-}\Gamma_h)/\Gamma$ and $\Gamma^2_{e^+e^-}/\Gamma$. For example, the use of formula (5) (and of the analogous expression for the cross section $\sigma^{\mu^+\mu^-}$) in the case of the $\psi(3105)$ resonance[1] yields $(\Gamma_{e^+e^-}\Gamma_h)/\Gamma=4.8\pm0.3$ and $\Gamma^2_{e^+e^-}/\Gamma=0.3\pm0.06$. Another method, independent of the value of σ, of determining these quantities is to compare the areas under the experimental resonance curves with the results of integration of the theoretical formulas. The values obtained thereby for $(\Gamma_{e^+e^-}\Gamma_h)/\Gamma$ and $\Gamma^2_{e^+e^-}/\Gamma$ of $\psi(3105)$ are then the same. The form of the experimental curve (and particularly the value of the maximum) depends essentially on the value of σ. A comparison of the forms of the theoretical and experimental curves makes it possible, in principle, to determine Γ and the parameter λ.

4. Let us discuss several phenomena connected with the interference of the resonance and the background. The total cross sections σ^h and $\sigma^{\mu^+\mu^-}$ contain interfer-

ence terms only if $J^{PC}=1^{--}$. The quantity $\sigma^{\mu^+\mu^-}$ has a minimum at $-(W-M)_{\min}\approx(\gamma_e/2)[(1+\sqrt{1+(\Gamma/\gamma_e)^2})]\gamma_e$ $=(3/2\alpha)\Gamma_{e^+e^-}$. If $\Gamma\ll\gamma_e$, then $(W-M)_{\min}\approx-\gamma_e$ and the cross section at the minimum reaches half the experimental value. At $\sigma\gtrsim\gamma_e$ this effect becomes very strongly smeared out by averaging over the energy spread. Without allowance for this spread, the position of the maximum in the cross section $\sigma^{\mu^+\mu^-}$ shifts first to the right by an amount $\sim(\Gamma^2/2\gamma_e)[1+\sqrt{1+(\Gamma^2/\gamma_e)}]^{-1}$. If $\Gamma\ll\sigma$, then the shift after averaging is $(W-M)$ $\sim\sqrt{2/\pi}(\Gamma\sigma/\gamma_e)$ (at $\Gamma<\gamma_e$). For $\psi'(3695)$ we have $\Gamma_{\psi'}>500$ keV, $\Gamma_{e^+e^-}\approx2-3$ keV,[2] and the corresponding shift is ~1 MeV. In the reaction $e^+e^-\to e^+e^-$, owing to the exchange diagram, resonances with arbitrary J can interfere with the background. At $J=0$ or 1 $(0<\pi/2)$, the interference is constructive to the left of the resonance. The position of the maximum should then shift to the left. At $J\geqslant2$ this situation obtains for small scattering angles. On going to large angles θ, the interference to the left of the resonance becomes destructive, since the angular harmonics describing the resonance process reverse sign in the angle range $0\leqslant\theta\leqslant\pi/2$.

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[1]J. E. Augustin *et al.*, Phys. Rev. Lett. **33**, 1406 (1974); J. J. Aubert *et al.*, Phys. Rev. Lett. **33**, 1404 (1974), C. Bacci *et al.*, Phys. Rev. Lett. 33, 1408 (1974).
[2]G. S. Abrams *et al.*, Phys. Rev. Lett. (to be published).
[3]V. N. Bayer and V. S. Fadin, Phys. Lett. **27B**, 233 (1968).
[4]G. Bonneau and F. Martin, Nucl. Phys. **B27**, 322 (1971).
[5]I. S. Gradshteĭn and I. M. Ryzhik, Tablitsi integralov, summ, ryadov i proizvedenii (Tables of Integrals, Sums, Series, and Products), Moscow 1962 [Academic, 1966].