Quasistable states in the physical spectrum of the dual model at large masses

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Physical states (PS) of a vertex, the connections of which with arbitrary other states are anomalously small if their mass is large enough, were observed in the dual spectrum. Such PS should therefore have small partial and total widths in comparison with the usual ones. We shall henceforth call these quasistable (QS) states.

In the generalized Veneziano model under the condition $\alpha(0)=1$, quasistable states are first observed on the second daughter trajectory (DT). [1] We shall seek a state of this type in the range $\alpha' M^2 > J \gg 1$, where J is the spin of the state. It is easily seen that the physical states that are quasistable are those which coincide, in the principal order in α, with the spurion states, i.e., $|F_{qs}\rangle = |S\rangle + (1/M)|\Phi\rangle$, where $|F_{qs}\rangle$ is a quasistable state, $|S\rangle$ is a spurion state, and $|\Phi\rangle$ is a certain Fock state with $\langle\Phi|\Phi\rangle \sim 1$. We take all the physical states (including the quasistable states) to be orthonormal.

The spurion states $|S\rangle$ are orthogonal to the physical states: $|S_m(q)\rangle = L^+_m(-q)|\Phi_{n-m}\rangle$, where q is the momentum of the state and L_m are Virasoro generators. [2] The complete set of independent spurion states $|S_m\rangle$ was constructed in the paper by Brower and Thorn. [3]

We can propose a method for explicitly constructing the quasistable states. We construct physical states satisfying the conditions

$$< F_n | L^+_1(-q) = 0\,; \qquad < F_n | L_0 = 1; \quad 1 + \frac{q^2}{2} = n, \tag{1}$$

$$< F_n | L^+_2(-q) = 0, \tag{2}$$

$$< F_n | S^{(0)}_1(-q) = 0 \qquad S^0_1(q) = L^+_1(-q) | F_{n-1} > \tag{3}$$

and coinciding in principal order in M (or q) with the suprion states. Starting with the fifth daughter trajectory, the conditions (1) and (2) turn out to be interdependent. Satisfying the conditions (1)—(3), we obtain the quasistable states $|F_{qs}\rangle$ on the second DT:

$$< F_{KC}| = \left[< f_0 \Big| L_2 + \frac{3}{2} L_1^2 + \frac{D-26}{2(q^2+D-3)} \tilde{a}_1^2 + \lambda < S^0_1 \Big| \right] \frac{1}{\sqrt{N}}, \tag{4}$$

where $\tilde{a}_{1\mu} = a_{1\mu} - (a_1 q) q_\mu / q^2$, and $\langle f_0|$ is a physical state on the principal trajectory. As usual, any physical state is defined accurate to the spurion state $|S_1^{(0)}\rangle$ with zero norm. The coefficient λ is obtained from the condition (3). From the construction of the state (4) we see that in the critical dimensionality ($D_{cr}=26$ for the given model) the quasistable states are transformed into spurion states with zero norm $\langle f_0 | L_2 \equiv \langle f_0 | (L_2 + 3L_1^2/2)$. [4] This quasistable state is the only one on the second DT.

Examining the construction of quasistable states on the third DT, we see that constructing the scheme becomes somewhat more complicated. In the case of the second DT, the state $|f_{0n}\rangle$ was an eigenfunction of the operator $\tilde{a}_1^2 L_2^+(-q)$. On the third DT we take as the increment to the spurion state $\langle f_1 | \tilde{a}_1^2$, and the following conditions are satisfied: $\langle f_1 | L_1^+(-q) = \langle f_1 | L_2^+(-q) = 0$ and $\langle f_1 | L_0 = -1$. We can see that $\langle f_1|$ is no longer an eigenfunction of the operator $\tilde{a}_1^2 L_2^+(-q)$. Therefore, in order to satisfy the condition (2), it is necessary to use two independent spurion states $\langle f_1 | L_2'$ and $\langle f_0 | L_3'$, for which the condition (1) is satisfied:

$$< f_1 | L'_2 = < f_1 | (L_2 + \frac{3}{2} L_1^2) \quad < f_0 | L'_3 = < f_0 | (L_3 + L_1 L_2 + \frac{1}{2} L_1^3). \tag{5}$$

Then, taking into account the commutation of the operators $\tilde{a}_1^2$ and $L_1^+(-q)$, we can obtain a physical state that coincides with the spurion state in the principal order in $a' q^2 \gg 1$:

$$< F'_{qs} | = \frac{1}{\sqrt{N}} \left[< f_1 | L'_2 + \frac{(D-26)\sqrt{q^2-4}\, q < f_0}{2(D-28)(q^2+2D-10)} L'_3 + \frac{(D-26)}{q^2+2D-10} < f_1 | \tilde{a}_1^2 \right]. \tag{6}$$

Satisfying the condition (3), we obtain the quasistable state $\langle F_{qs}| = \langle F'_{qs}| + \sum_{i=1}^3 \lambda_i S^{(0)}_{1i}$, where $S^{(0)}_{1i} = \langle f_{2i} | L_1(q)$ and $\langle f_{2i}|$ is the physical state of the second DT off the mass shell. It follows from the explicit form of (6) that at $D=26$ we obtain a spurion state with zero norm from the initial quasistable state.

On the fourth DT, to satisfy the condition (2) it is necessary to use six independent spurion states $\langle f_{2i} | L_2'$, $\langle f_1 | L_3'$, and $\langle f_0 | L_4^j$; where $i=1, 2, 3$ and $j=1, 2$, since the state $\langle f_{2i} | \tilde{a}_1^2 L_2^+(-q)$ of the second DT contains six possible Fock states.

We next satisfy the condition (3) and construct three quasistable states orthogonal to one another. It is easily understood that the coefficients of the increments vanish at $D=26$. Indeed, the coefficient of $\langle f_2^i | \tilde{a}_1^2$ is chosen from the condition that the coefficients r_σ be equal to zero:

$$\left(\sum_{i=1}^{3} a_i < f_{2i} | L'_2 + \beta < f_{2j} | \tilde{a}_1^2 \right) L_2^+(-q) = \sum_{j=1}^{3} r_\sigma < f_{2\sigma} \Big| + \sum_k \gamma_k < S_k \Big| .$$

However, when the states $\langle f_{2i} | L_2'$ are acted upon by the operator L_2^+ we obtain the factor $(D-26)$, which determines the vanishing of the coefficient β at a dimensionality $D_{cr}=26$. To determine the coefficients in the other terms of the expression

$$\langle F| = \langle f_2^j | L_2^{\prime} + \sum_{i \neq j} a_i \langle f_{2i} | L_2^{\prime} + \gamma \langle f_1 | L_3^{\prime} + \sum_j \delta_j \langle f_0 | L_4^{(j)} + \beta \langle f_{2j} | \tilde{a}_1^2 \qquad (7)$$

we use a system of equations whose inhomogeneous terms are proportional to $(D-26)$. Therefore all the coefficients β, γ, and δ_j vanish at $D=26$.

For the construction of the quasistable states on the fifth DT we have one condition of the type (1), which coincides with a condition of the type (2). Therefore 12 spurion states satisfying the condition (1) are sufficient on the fifth DT. In the physical case $D=4$, there exist on the fifth DT five quasistable states. On the sixth DT there will be already 11 such states. We note that we are considering only states with positive internal parity. It is easily seen that the number of the spurion states satisfying the condition (1) is sufficient for the construction of the QS on any DT, and the spin of the state is $J \gg 1$. Indeed, the number of spurion states satisfying the condition (1) is equal to $N_S(k) - N_S(k-1)$, where $N_S(k)$ is the number of the spurion states on the kth DT. The number of independent conditions of type (2) is also $N_S(k) - N_S(k-1)$, i.e., it coincides exactly with the number of the possible combinations of the spurion states $\langle f | L_m$. Therefore the number of quasistable states on the kth DT is equal to the number of physical states on the $(k-2)$nd DT, $N_{qs} = N_{FD_{k-2}} = N_{\Phi D_{-1\,k-2}}$, and the asymptotic estimate for the number of Fock states at large values of the mass of the pole takes the form

$$N_{\Phi Dk} \sim C k^{-B} \exp(2\sqrt{Dak}), \qquad (8)$$

where $a = (\pi^2/16)B = (D+2)/4$. We see that the number of quasistable staies coincides with the number of the type $\langle f(k-2) | L_2'$. At $D=26$, the spurion states $\langle f_{k-2} | L_2'$ become spurion states with zero norm, and therefore drop out of the spectrum together with their conjugate states $\langle f_{k-2}^{(-q)} | L_2'(-q)$, in analogy with the states $\langle f_{k-1} | L_1$ and $\langle f_{k-1}^{(-q)} | L_1^{(-q)}$. Our quasistable states are transformed at $D=26$ into the spurion states $\langle f_{k-2} | L_2'$, and therefore drop out of the dual amplitude not asymptotically but exactly. Thus, quasistability is a manifestation of spurion states with zero norm in the case of lower dimensionalities $D<26$. From a direct calculation of the quasistable states on the second, third[1] and fourth DT we can assume that there exist no other constructions of quasistable states at large masses. However, we can construct physical states under the conditions $J \sim 1$ and $a'q^2 \gg 1$, which are quasistable with respect to the number of trajectory $k \gg 1$. For example, $\langle F| \sim [(D-26)/(k+c)] \langle 0 | (a_{15}^2 - a_{16}^2)^{k/2-2} (a_{15}^2 + a_{16}^2) + \sum_j a_j \langle S_j |$ for the kth DT at $D \geq 6$, where a_j and c are constants. These states do not exist at the physical dimensionality $D=4$.

It is of interest to attempt to observe experimentally heavy quasistable states. Such a possibility is considered, e.g., in[5]. Worthy of special attention in this connection is the experimental observation, in e^+e^- annihilation, of anomalously narrow resonances with relatively large masses, $M_\psi^2 \sim 10$ GeV^2 and $M_{\psi'}^2 \sim 13.7$ GeV^2. The experimental observation of quasistable resonances is important for the separation of dual models with optimal D, for if quasistable states do not exist, then the critical dimensionality of the model required is $D=4$. On the other hand, if they do exist, then it is possible to use models with $D_{cr} > 4$ as approximations to the physical models.

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