Model of asymptotically free massive particles

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A procedure is proposed for constructing theories of asymptotically free massive particles. The general scheme is illustrated with a concrete example.

1. In this article we propose a procedure for the construction of asymptotically free theories with massive particles, which consist in the following. We first choose the Lagrangian L describing the interaction of gauge, spinor, and scalar fields, which has an additional symmetry (on top of gauge symmetry) which leads to asymptotic freedom. Specifically, we consider a Lagrangian[1] having gauge symmetry and supersymmetry.[2] The additional symmetry can lead to the impossibility of the Higgs mechanism (in the Lagrangian chosen by us, all the particles are massless). The next step is to add to L mass terms that are compatible with gauge symmetry, but which violate the additional symmetry. The mass terms for the scalar field must be chosen in such a way that spontaneous breaking of gauge symmetry takes place, so that *all* the particles become massive. The asymptotic freedom is then preserved in the theory. This fact follows from the statement that all the renormalization constants do not depend on the dimensional parameters of the Lagrangian. This state-

ment is formulated more accurately in the next section. In Sec. 3 we describe in greater detail a model illustrating the proposed procedure.

2. We consider a renormalizable-theory Lagrangian in the general form

$$L = \Sigma(\bar{\Gamma}_i^{(4)} + M_i^2 \Gamma_i^{(2)} + m_i \Gamma_i^{(3)} + g_i \Gamma_i^{(4)}), \tag{1}$$

where the vertices are classified by dimensionality: $\bar{\Gamma}^{(4)}$ are kinetic terms of the type $\partial_\mu \phi \partial^\mu \phi$ and $\bar{\psi}\gamma^\mu \partial_\mu \psi$, $\Gamma_i^{(2)}$ are mass terms of boson fields, $\Gamma_i^{(3)}$ are mass terms of fermion fields and vertices of the type ϕ^3 and $\phi\partial\phi$, while $\Gamma_i^{(4)}$ are vertices with dimensionless coupling constants of the type ϕ^4, $\bar{\psi}\psi\phi$, and $\phi^2\partial\phi$. The following statement holds true: it is possible to find functions $Z_i(g, \Lambda/\lambda)$ that depend only on the dimensionless parameters, and also on the ratio of the cutoff parameter Λ to a certain mass parameter λ (normalization point), so that the theory described by the (renormalized) Lagrangian

$$L_R = \Sigma[\tilde{Z}_i^{(4)} \bar{\Gamma}_i^{(4)} + (M_i^2 Z_{ij}^{(2)} + m_i m_k Z_{ikj}^{(2)} + \lambda^2 Z_j^{(2)})\Gamma_j^{(2)} + m_i Z_{ij}^{(3)} \Gamma_j^{(3)} + g_i Z_{ij}^{(4)} \Gamma_j^{(4)}], \tag{2}$$

is finite. The parameters M_i, m_i, g_i, and λ in (2) are assumed to be finite. We note that the physical masses do not coincide with M_i or m_i, but are finite functions of these parameters. The proof of this statement[1)] is presented with the aid of a generalization of the scheme of a proof proposed by Weinberg,[3] of an analogous statement for theories without scalar particles (but without resorting to the massless theory).

3. We consider a theory illustrating the general scheme described in Sec. 1. The Lagrangian is given by

$$L = L_{ss} + L_m . \tag{3}$$

L_{ss} describes asymptotically free supersymmetry and $SU(2)$ gauge-invariant theory[1] in a special gauge:

$$L_{s.s.} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \frac{1}{2} (\nabla_\mu^{ab} \tilde{\phi}_b)^2 + \frac{1}{2} (\nabla_\mu^{ab} \Lambda_b)^2 - \frac{g^2}{2} (\tilde{\phi}^2 \Lambda^2 - (\tilde{\phi}\Lambda)^2) + i\bar{\psi}^a \gamma^\mu \nabla_\mu^{ab} \psi^b + ig\epsilon^{abc} \bar{\psi}^b (\tilde{\phi}_a + i\gamma_5 \Lambda_a) \psi^c - \partial^\mu C^{+a} \nabla_\mu^{ab} C^b ; \tag{4}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu ; \quad \nabla^{ab}_\mu = \partial_\mu \delta^{ab} + g\epsilon^{acb} A^c_\mu ; \tag{5}$$

A^a_μ, ψ^a, $\tilde{\phi}_a$, and Λ_a are triplets of the gauge, spinor, scalar, and pseudoscalar fields, the gauge is $\partial^\mu A^a_\mu = 0$, and C^a is a triplet of fictitious Fermi scalars.

The mass terms are chosen in the form

$$L_m = - M\bar{\psi}\psi - \frac{1}{2} m^2 \Lambda^2 + \frac{1}{2} \mu^2 \tilde{\phi}^2 \tag{6}$$

L_m preserves the gauge invariance, but violates the supersymmetry. According to Sec. 2, the theories (3) and (4) have the identical divergences of the vertices and wave functions, so that the theory (3) is (multiplicatively) renormalizable and asymptotically free, just as theory (4). We note that the theory (3) has the same form as the usual Yang-Mills theory in the gauge $\partial^\mu A^a_\mu = 0$.

The incorrect sign in front of $\tilde{\phi}^2$ in (6) enables us to realize the spontaneous breaking of the symmetry. We introduce a new field ϕ_a:

$$\tilde{\phi}_a = \begin{pmatrix} \xi \\ 0 \\ 0 \end{pmatrix} + \phi_a , \qquad \langle 0 | \phi_a | 0 \rangle = 0 , \tag{7}$$

and substitute this expression in the Lagrangian. We write out the terms of interest to us:

$$\frac{1}{2} \partial_\mu \phi_a \partial^\mu \phi_a + \frac{1}{2} \mu^2 \phi^2 + \mu^2 \xi \phi_1 . \tag{8}$$

The remaining terms either do not contain the field ϕ, or are more than quadratic in the fields. The field A^1_μ remains massless (photon), while the fields $A^{2,3}_\mu$ acquire a bare mass $M_0 = g\xi$, which is chosen as the independent parameter in place of μ^2. The parameter μ^2 should be chosen from the condition $\langle 0 | \phi_a | 0 \rangle = 0$. Let us consider this condition in the zeroth approximation in g. From (8) we obtain

$$\mu^2 \xi = \frac{\mu^2}{g} M_0 \Big|_{g=0} = 0, \tag{9}$$

from which (under the condition $\xi \neq 0$) it follows that μ^2 takes the form

$$\mu^2 = g^2 a , \tag{10}$$

and the term linear in ϕ_a takes the form

$$g a M_0 \phi_1 . \tag{11}$$

It is now perfectly clear that in each order of perturbation theory it is possible to satisfy the condition $\langle 0 | \phi_1 | 0 \rangle$ by a corresponding (unique) choice of a.

It is necessary next to verify the stability of the theory, i.e., to establish the sign of the square of the mass in the field ϕ_1 (the fields $\phi_{2,3}$ are unphysical, and the mass in the remaining fields is normal). Calculation in second order in g yields (with allowance for the expression obtained for a from the condition $\langle 0 | \phi_1 | 0 \rangle = 0$ in first order in g):

$$\Pi(0) = \frac{4g^2}{(2\pi)^4} \int d^4p \left[\frac{2(M - M_0)^2}{(p^2 + (M - M_0)^2)^2} + \frac{2(M + M_0)^2}{(p^2 + (M + M_0)^2)^2} - \frac{3M_0^2}{(p^2 + M_0^2)^2} - \frac{M_0^2}{(p^2 + M_0^2 + m^2)^2} - \frac{4M^2}{(p^2 + (M - M_0)^2)(p^2 + (M + M_0)^2)} \right], \tag{12}$$

where the integral is Euclidean, and Π is defined by

$$(S_{\phi_1}(q))^{-1} = -i(q^2 - \Pi(q^2)), \quad q^2 = q_0^2 - \mathbf{q}^2 \tag{13}$$

It is seen that $\Pi(0)$ is finite (as expected on the basis of (Sec. 2). In addition, there exists a region of values of the parameters (e.g., $M_0 \gg M$, $m^2 > 12 M^2$), when $\Pi(0)$ is positive. We note that a is negative in this case.

Thus, by choosing the parameters M_0, m, and M we can ensure a normal mass of the field ϕ_1, i.e., stability of the theory. We note that at $M = 0$ the theory is always stable, and one of the fermions has zero mass (neutrino).

If the theory (3) is regarded only as a gauge theory, then we can change over to the gauge $\phi_2 = \phi_3 = 0$ (with corresponding modification of the additional vertices). In this gauge, the effective potential in the approximation in g^2 (with allowance for the radiative corrections), is equal to ($i = 2, 3$):

$$V = \frac{1}{2}\,\Pi(0)\,\phi_1^2 + \frac{1}{2}\,(m^2 + \delta m_1^2)\,\Lambda_1^2 + \frac{1}{2}\,(m^2 + M_0^2 + \delta m_2^2)\,\Lambda_i^2$$

$$+ g M_0\,\phi_1 \Lambda_i^2 + \frac{g^2}{2}\,\phi_1^2\,\Lambda_i^2 \quad , \qquad (13)$$

where δm_1^2 and δm_2^2 are radiative corrections proportional to g^2 to the masses of the Λ_a fields. It is easy to verify that at $\Pi(0) > 0$ the potential (13) has a unique minimum at the point $\phi_1 = \Lambda_a = 0$.

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[1)]This statement was also proved by another method by Collins and Macfarlane.[4]

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