Observation of Feigenbaum period-doubling bifurcations in an MHD instability of an electron-hole plasma in bismuth

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As the direct current through a bismuth sample is increased, the carrier flux becomes unstable, and the period of the electromagnetic waves emitted by the sample goes through a sequence of doublings. The observed results agree with Feigenbaum's model.

When a direct current is passed through a bismuth sample at liquid-helium temperature, the flux of current carriers becomes unstable at a certain threshold current. The instability is manifested in oscillations in the magnetic field.[1] Even before the instability occurs, the magnetic field produced by the current and the magnetoresistance cause a nonuniform distribution of the current over the cross section of the sample (the current density is higher at the center than at the periphery). At the time of the instability, the value of the parameter $\omega_c \tau$ reaches 20–30 at the periphery (ω_c is the cyclotron frequency, and τ the carrier relaxation time), so that the current (and thus the magnetic field) is highly uniform. The instability is described by a system of equations consisting of the constitutive equation $\mathbf{E} = \rho_0 \mathbf{j} + \rho_1 \mathbf{H} \times \mathbf{j} + \rho_2 \mathbf{H} \times \mathbf{H} \times \mathbf{j}$ (the first term describes the resistance, the second the Hall effect, and the third the magnetoresistance) and Maxwell's equations.[1,2] The instability results from the effect of the current distribution over the cross section of the sample on the magnetic field distribution, which in turn affects the current distribution.

In this letter we report a study of the initial stage of the instability. The experimental configuration is the same as in the previous experiments.[1] The output signal from a coil at the center of the high-quality ($\rho_{300}/\rho_{4.2\mathrm{K}} \approx 600$), bulk single-crystal sample ($l \approx 100$ m, diam ≈ 15 mm) is amplified and sent to an analog-to-digital converter to a computer. The apparatus digitizes (within $\sim 0.1\%$) and stores 2048 successive readings of the signal (the time interval between readings is 512 μs), carries out a Fourier analysis, and sends both the signal and its spectrum to a plotter.

Figures 1 and 2 show some representative signals and spectra for various values of the direct current through the sample. The records at the top correspond to the threshold current for the instability. The nearly sinusoidal signal that arises at the threshold for emission is unstable, as can be seen from the time variation of its amplitude (Fig. 1) and thus the presence of a noise amounting to $\sim 1\%$ of the amplitude of the fundamental frequency (Fig. 2). As the current is raised, new components appear in succession in the spectrum, with frequencies $f_0/2, f_0/4, f_0/8$, etc., along with their harmonics. In other words, there is a sequential doubling of the period of the observed oscillations. Although the fundamental frequency in these experiments depends on a parameter—the current through the sample (Fig. 2; this is a common property of nonlinear systems)—the new subharmonics and their harmonics that arise as the current is raised

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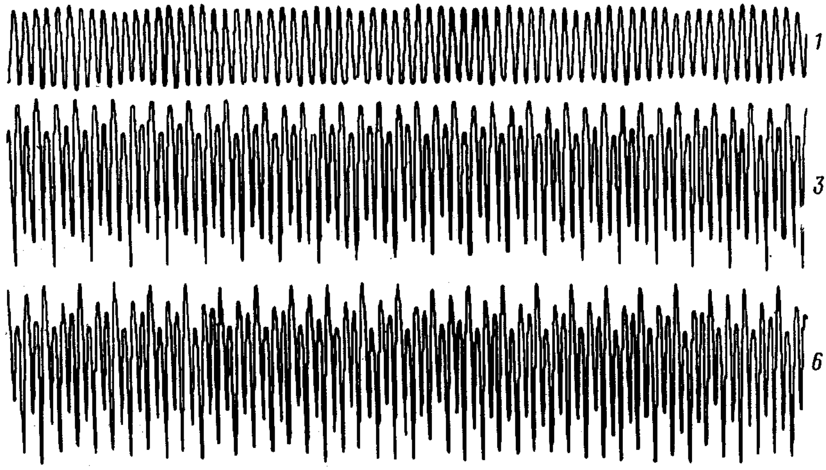

FIG. 1. "Oscilloscope traces" of the signals for three values of the direct current through the sample, corresponding to the spectra in Fig. 2 with the same labels. $T = 1.5$ K.

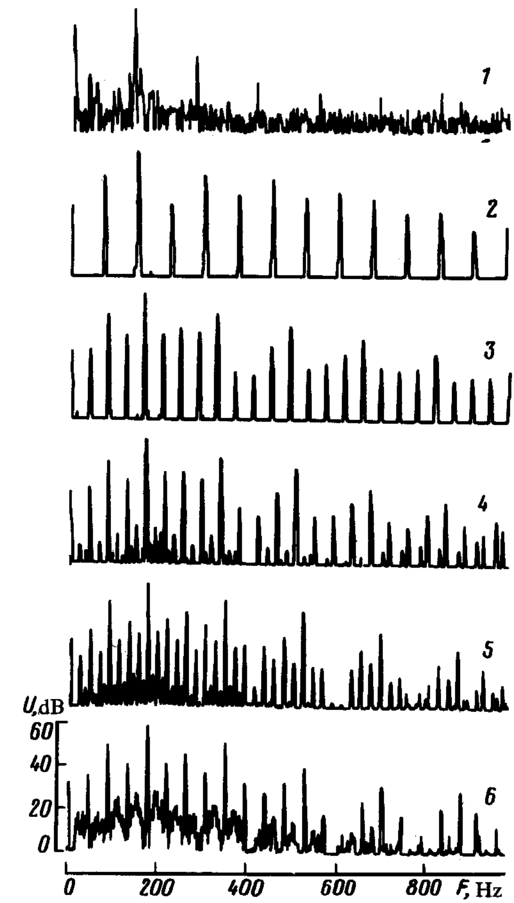

FIG. 2. Spectra of the voltage from the coil at the following values (in amperes) of the direct current through the sample: 1—32.96; 2—38.08; 3—40.30; 4—41.08; 5—41.67; 6—41.91.

TABLE I

k	4	6	8	9	10	11	12	13	15	16
f, Hz	21,52	32,93	43,09	48,93	53,68	58,17	64,62	72,49	79,16	86,18
$f_0/32$, Hz	21,54	32,31	43,09	48,47	53,86	59,24	64,63	70,01	80,78	86,17
Δ, Hz	0,02	−0,62	0,00	−0,46	0,18	1,07	0,01	−2,48	1,62	−0,01

have frequencies $f_0k/2^n$, where the frequency f_0 corresponds to the altered value of the current. This circumstance is evident from Table I, which shows the emission frequencies for a current of 41.67 A found through a quadratic interpolation of the crest of each peak (spectrum 5 in Fig. 2), the frequencies calculated from the formula $f_0k/32$ for certain values of k (the fundamental frequency $f_0 = 172.34$ Hz corresponds to the same current), and their deviations Δ. We see from Table I and Fig. 2 that the period doubles five times as the current is raised, in accordance with the appearance of multiples of the frequency $f_0/32$ in the spectrum of harmonics. This circumstance indicates that the instability that we have observed arises in accordance with Feigenbaum's universality theory[3] which describes the transition from a regular behavior of a system to the initial stage of turbulence. According to this model, the road to chaos in many nonlinear systems as some parameter is changed monotonically involves period-doubling bifurcations. In other words, at certain (bifurcation) values of the parameter, the previously stable cycle gives way to a stable cycle with twice the period. This doubling continues to infinity; the sequence of bifurcation values of the parameter converges on a certain limit, beyond which the behavior is chaotic.

We did not determine the values of the current at which the successive subharmonics arise in the spectrum in the present experiments, so that we cannot determine the rate at which the sequence converges. Nevertheless, as can be seen from the Fig. 2 caption, the differences between successive values of the current at which new subharmonics appear in the spectrum decrease; i.e., as n increases, the regions in which the 2^n cycles exist along the scale of the parameter become smaller. This result is qualitative confirmation that the road to chaos in our case is as described by this model.

We see from Fig. 2 that the amplitudes of the subharmonics (and of their harmonics) which arise with increasing current reach certain values and then become constant as the current is raised further (e.g., the amplitude of the $f_0/2$ subharmonics in spectra 2–6 is constant within $\sim 1\%$). We also see that the amplitudes of the components f_0, $f_0/2$, $f_0/4$, ... (and of their harmonics) become systematically smaller. Both of these facts agree with the theory of universality.

It can be concluded from the various results observed here that the onset of the instability in these experiments is described by Feigenbaum's scenario. It would clearly be interesting to experimentally determine the universal constants describing the rate of convergence of the sequence of bifurcation values of the parameter and the ratio of amplitudes in the spectrum. We are presently trying to solve this problem.

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