Coherent structures in a Heisenberg anisotropic array

Ya. I. Granovskiĭ and A. S. Zhedanov
Donetsk State University

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The wave function of the coherent state of the *XYZ* array of arbitrary spins is constructed explicitly. This function describes the periodic structure which generalizes the domain-wall solution.

We consider a one-dimensional array of spins coupled by an exchange interaction

$$H = -\sum_n J_{\alpha\beta} S_n^\alpha S_{n+1}^\beta \ . \tag{1}$$

The tensor $J_{\alpha\beta}$ is assumed to be completely anisotropic, i.e., in the major axes $0 < J_1 < J_2 < J_3$. We seek the steady-state solutions of the Schrödinger equation in the form of a coherent structure [for Hamiltonian (1) states of this sort were considered by Feynman[1] and Pokrovskiĭ and Khokhlavech[2]]:

$$\psi = \prod_n R_n \,|0\rangle , \tag{2}$$

where R_n is the operator of the n-th spin rotation, and $|0\rangle$ is a state in which each spin has a maximum projection along the z axis corresponding to J_3.

Since

$$R_n^+ S_n^\alpha R_n = R_n^{\alpha\gamma} S_n^\gamma , \tag{3}$$

we can reduce the Schrödinger equation to the form

$$\widetilde{H}\,|0\rangle = \epsilon\,|0\rangle, \qquad \widetilde{H} = -\sum_n \widetilde{J}^n_{\alpha\beta} S_n^\alpha S_{n+1}^\beta \ , \tag{4}$$

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where

$$\tilde{J}^{n}_{\alpha\beta} = J_{\lambda\mu} R^{\lambda\alpha}_{n} R^{\mu\beta}_{n+1} \quad . \tag{5}$$

Equation (4) is valid if the conditions

$$\tilde{J}^{n}_{xx} - \tilde{J}^{n}_{yy} + i\tilde{J}^{n}_{xy} + i\tilde{J}^{n}_{yx} = 0, \quad \tilde{J}^{n}_{xz} + \tilde{J}^{n-1}_{zx} + i\tilde{J}^{n}_{yz} + i\tilde{J}^{n-1}_{zy} = 0 \tag{6}$$

are satisfied.[2] Here

$$\mathcal{E} = -\sigma^2 \sum_n \tilde{J}^{n}_{zz} \quad . \tag{7}$$

Conditions (6), quadratic recurrence relations with respect to the components $R^{\alpha\beta}_{n}$, are satisfied by Jacobi elliptic functions with a modulus $k\,(k' = \sqrt{1-k^2})$:

$$R^{\alpha 3}_{n} = (\xi\, \mathrm{dn}\, t, \;\; \eta \cos n\, t \;\; \zeta \sin n\, t), \tag{8}$$

where

$$t = \omega n + \omega_0, \quad \eta = \sqrt{1-\xi^2}, \quad \zeta = \sqrt{1-k'^2\xi^2}, \tag{9}$$

$$\cos n\, \omega = J_1/J_3, \quad k^2 = (J_3^2 - J_2^2)(J_3^2 - J_1^2)^{-1} . \tag{10}$$

Since the components $R^{\alpha 1}_{n}$ and $R^{\alpha 2}_{n}$ can be dropped from conditions (6), we will not write them out in explicit form.

The "integration" constants ω_0 and ξ determine the position and magnitude of the maximum of the components $R^{\alpha 3}_{n}$; however, the energy density ϵ per array site does not depend on ξ and ω_0:

$$\epsilon = -\sigma^2 \left\{ \frac{J_1 J_2}{J_3} + \sqrt{J_3^2 - J_1^2} \left[E(\omega) - \frac{\omega E}{K} \right] \right\} , \tag{11}$$

where $E(\omega)$ is an incomplete elliptic integral, and E and K are type-2 and type-1 complete elliptic integrals with modulus k.

The solution of (8) is valid for any value of the spin σ. From Eq. (11) we see that this class of states is an extension to an arbitrary spin of the so-called generating state[3] found by Baxter for spin 1/2.

On the other hand, state (8) is a quantum analog of a classical solution of the band-domain-structure type with a period $4K/\omega$ (for $\xi = 0$). For $\xi \neq 0$ the spin has a nonzero average projection of $\pi\sigma\xi/2K$ along the x axis.

We note that for any spin the main characteristics (energy, correlations) are the same as those for the corresponding classical solution. This is as it should be, since (2) are the coherent spin states whose properties are most closely related to the classical properties.[4]

In the case of a uniaxial anisotropy ($J_1 = J_2$), the solution of (8) acquires the shape of a Landau-Lifshitz domain wall,[5,6]

$$(R_n^{\alpha 3})_{k=1} = (\xi \cosh^{-1} t,\ \ \eta \cosh^{-1} t,\ \ \tanh t) \tag{12}$$

with the energy[6]

$$\mathcal{E}_{\mathrm{DW}} = 2\sigma^2 \sqrt{J_3^2 - J_1^2}\,.$$

The degeneracy of ξ (the energy indistinguishability of the Bloch and Néel walls) in this case is governed by the 0(2) invariance of the XXZ model.

The solution of (8) raises an interesting possibility of finding other exact solutions for the XYZ array with an arbitrary spin of energy which is different from (11). The feasibility of these solutions should be studied separately.

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