Nonlinear drift tearing mode: Hard onset and stabilization mechanisms

A. A. Galeev, L. M. Zelenyĭ, and M. M. Kuznetsova
Institute of Cosmic Studies, Academy of Sciences of the USSR

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A nonlinear theory is derived for the development of magnetic perturbations in collisionless configurations with magnetic shear. The dynamics of the ions and of the electrostatic fields, which determine the stabilization of the mode, is incorporated in the analysis of the evolution of the instability. The drift tearing mode is found to have metastable properties: In the nonlinear stage, even perturbations of finite amplitude which are stable in the linear stage may grow.

The onset of magnetic perturbations determines the conditions under which a plasma can be confined by a magnetic field in the laboratory and in space. The nonlinear evolution of magnetic islands therefore remains a topic of major interest to researchers. Magnetic islands form near so-called singular surfaces at which the longitudinal component of the perturbation wave vector vanishes, $k_{\parallel}(X_S)=0$, by virture of the free energy associated with the magnetic shear. In the linear case,[1] which is the simplest case, the instability (the tearing mode) results from a resonant absorption of the energy of the perturbations by electrons in a small neighborhood ($|x|=|X-X_S|<\delta_e$) of the singular surface. The thickness of this neighborhood, δ_e, can be found easily by noting that within this neighborhood the Doppler shift $k_{\parallel}(x)v_{Te}$ must be small in comparison with the perturbation frequency ω:

$$\delta_e=\omega\Big/\left(\frac{dk_{\parallel}(x)}{dx}v_{Te}\right)\Big|_{x=0}.$$

The primary effect in a study of the nonlinear dynamics of magnetic perturbations is the modification of the orbits of resonant particles (electrons) caused by these perturbations.[2–4] The nonlinear perturbation of electron trajectories was studied most systematically in a recent paper by Hazeltine and Swartz.[5] Their final result, however, turns out to be the same as was found in some simpler calculations[2,3]: In the collisionless case, the instability is stopped when the half-width (w) of a magnetic island becomes comparable to δ_e. In fact, it is impossible to find any other result if we consider electrons exclusively. It has been shown[4,6] that when the motion of ions is taken into account the nonlinear drift tearing mode

$$\left(\omega\sim\omega_e,\qquad \omega_j=-\frac{ck\,[T_j\vec{\nabla}n\mathbf{B}]}{e_jB^2n(x)}\right)$$

may develop even under the condition $w\gg\delta_e$. In Refs. 4 and 6, however, the electrostatic component of the perturbed electric field, $ik_{\parallel}\phi(x)$, was ignored; this component actually restricts the region of the interaction, $E_{\parallel}(x)\neq 0$, to a characteristic distance[7,8] $|x|<\delta_{\phi}$.

The behavior of $\phi(x)$ is determined primarily by the ion dynamics, so that linear expressions for $\phi(x)$ are legitimate even under the condition $w > \delta_e$. Incorporating the ion motion gives rise to an additional expenditure of energy on the excitation of plasma motions of the ion acoustic type. In the linear case, this effect may result in a complete stabilization of short-wave perturbations.

As we show below, in the nonlinear stage, with $w > \delta_e$, in which the leading role is played by trapped rather than resonant electrons, the mode changes (analogs of these modes in a toroidal geometry are the trapped-particle modes studied in Ref. 9). There is a corresponding change in the role of the interaction with longitudinal motions of the ions. For a nonlinear mode, this interaction has a destabilizing effect, so that, in particular, perturbations of finite amplitude which are stable in the linear approximation may grow (the "hard" onset of an instability). The amplitude of the nonlinear mode increases as a power function of the time, until it reaches a certain level determined by the condition $w \lesssim w_{\rm sat} \sim \delta_\phi$.

As the initial configuration with magnetic shear, we select the generalized Harris distribution

$$\mathbf{B} = B_0 \,\mathrm{th}\frac{X}{L}\,\mathbf{e}_z + B_y \mathbf{e}_y\,, \qquad L_S \simeq L B_y / B_0\,. \tag{1}$$

Let us examine the stability of the Fourier harmonics of perturbations of the vector potential $\mathbf{A}$ and of the scalar potential ϕ: $\{\mathbf{A},\phi\} = \{\mathbf{A}(x),\phi(x)\}\exp(-i\omega t + i\mathbf{kr})$, where $x = X - X_S$ and $\mathbf{k} = (0,k_y,k_z)$. For $A_\parallel(x)$ we have the familiar equation[8,9]

$$\frac{d^2 A_\parallel}{dx^2} - k^2 A_\parallel - V_0 A_\parallel = \sum_j V_j A_\parallel\,, \quad V_0 = -\frac{2k_z^2}{L^2 k^2}\,\mathrm{ch}^{-2}\left(\frac{X_S + x}{L}\right)\,, \tag{2}$$

$$V_j(x) A_\parallel = -\left(A_\parallel - \frac{k_\parallel c}{\omega}\phi\right) \times \frac{4\pi e_j^2}{c^2 T_j} \int_{-\infty}^{\infty} v_\parallel f_{0j}\, i(\omega - \omega_j)\, dv_\parallel \int_{-\infty}^{0} e^{-i\omega\tau + i\mathbf{kr}(\tau)} v_\parallel(\tau)\, d\tau. \tag{3}$$

The eigenvalues ω are determined in this case by the dispersion relation

$$\Delta' = L \sum_j \int_0^{\infty} V_j\, dx\,. \tag{4}$$

The parameter Δ' corresponds to the free energy of the instability.[1,4,6]

Zelenyĭ and Taktakishvili[6] derived equations for the trajectories of the particles in the perturbed magnetic field and showed that the integral in (4) can be broken up into three parts:

$$\sum_j \int_0^{\infty} V_j\, dx = \sum_j U_j^{TR} + \sum_j U_j^{S} + \sum_j U_j^{UT}\,, \tag{5}$$

where U_j^{TR} is the component of the "trapped" particles which are moving within an island along closed trajectories, U_j^S is the component from particles which are moving near the separatrix (the boundary of an island), and U_j^{UT} is the component from "untrapped" particles outside the island.

Let us calculate the untrapped-ion component U_i^{UT}, taking the structure of $E_\parallel(x)$ into account. If $w<\delta_i = |\omega/v_{Ti}(dk_\parallel/dx)|$, the trajectories of the untrapped ions are perturbed only slightly, and they can be described in the linear approximation.[6] As the drift tearing mode develops ($\omega=\omega_e+i\gamma, |\gamma|\ll|\omega|$), the behavior of $\phi(x)$ is determined primarily by ions. Accordingly, if $w\ll\delta_i$ and $\delta_i>\rho_i$, then we can again use the linear expression[8] for $\phi(x)$ in the region $|x|>w$:

$$\phi(x) = \frac{\omega A_\parallel}{k_\parallel c}\left[\frac{ix^2}{2\delta_\phi^2}\int_0^{\pi/2}\exp\left\{-\frac{ix^2}{2\delta_\phi^2}\cos\theta\right\}\sqrt{\sin\theta}\,d\theta\right], \quad \delta_\phi=\sqrt{\delta_i\rho_i}<\delta_i. \tag{6}$$

Carrying out the integration over the unperturbed ion trajectories in (3), and using (6), we find the total integral contribution of untrapped ions to (4):

$$U_i^{UT} = L\int_w^\infty V_i dx = \epsilon_i^{-2}\frac{\omega-\omega_i}{\omega}\frac{\delta_\phi}{L}(F_1-iF_2), \tag{7}$$

$$F_1 = \frac{\Gamma(3/4)\pi}{\sqrt{2}\Gamma(1/4)} - \bar{w} - \frac{\sqrt{\pi}\Gamma(3/4)2^{3/4}}{4}\int_0^{\bar{w}^2/2} z^{1/4}\mathbf{H}_{1/4}(z)dz, \tag{8}$$

$$F_2 = \frac{\Gamma(3/4)\pi}{\sqrt{2}\Gamma(1/4)}\left[1-\frac{\pi\bar{w}^2}{4}\left(J_{1/4}\left(\frac{\bar{w}^2}{2}\right)\mathbf{H}_{-3/4}\left(\frac{\bar{w}^2}{2}\right) - J_{-3/4}\left(\frac{\bar{w}^2}{2}\right)\mathbf{H}_{1/4}\left(\frac{\bar{w}^2}{2}\right)\right)\right], \tag{9}$$

where $\bar{w}=w/\delta_\phi$; $\epsilon_j = v_{Tj}m_jc/eB_0L \cong \rho_iB_y/LB_0$; $J_{1/4}$ and $J_{-3/4}$ are Bessel functions; and $\mathbf{H}_{1/4}$ and $\mathbf{H}_{-3/4}$ are Struve functions.

Since the electrons and also the trapped ions and the ions near the separatrix escape the effect of the decrease in $E_\parallel(x)$ due to the electrostatic field $ik_\parallel\phi(x)$, we can use in (4) the expressions found previously[6] for U_j^{TR}, U_j^S, and U_e^{UT}. Using (7), we find

$$\Delta' = \epsilon_i^{-2}\frac{\omega-\omega_i}{\omega}\frac{\delta_\phi}{L}(2\bar{w}+F_1-iF_2)+U_e, \tag{10}$$

$$U_e = \epsilon_e^{-2}\frac{\omega-\omega_e}{\omega}\frac{w}{L}\begin{cases}2-i\sqrt{\pi}(\delta_e/w), & w\ll\delta_e & (11)\\ 0{,}2-i\sqrt{\pi}(\delta_e/w)^3, & w\gg\delta_e. & (12)\end{cases}$$

The solution of Eq. (10) can be written in the form $\omega=\mathrm{Re}\omega+i\gamma, \mathrm{Re}\omega\sim\omega_e$:

$$\gamma(w) = \omega_e\frac{\epsilon_e^2 2L}{\delta_e\sqrt{\pi}}\frac{\Delta'-(\epsilon_i^{-2}F_1\delta_\phi/L)(1-2w/\sqrt{\pi}\delta_e)}{1+(2w/\sqrt{\pi}\delta_e)^2}, \quad w<\delta_e \tag{13}$$

$$\gamma(w) \cong \omega_e \frac{10\epsilon_e^2}{\epsilon_i^2} \frac{\delta_\phi}{w} F_2(w), \quad w > \delta_e . \tag{14}$$

With $w=0$ and $F_1(0)=0$, the growth rate in (13) reduces to the standard linear expression[1]: $\gamma_L = \omega_e(\epsilon_e^2 2\Delta' L/\delta_e\sqrt{\pi})$. Correcting for the longitudinal motions of the ions, we find that γ_L decreases, and at $a=\epsilon_i^{-2}\delta_\phi F_1(0)/L\Delta' > 1$ the mode is completely stabilized.[7,8] The value of a thus determines how far the mode will be from the threshold for linear stability. Figure 1 shows curves of γ versus w calculated from (10)–(14) for various values of a.

The development of drift tearing modes in the nonlinear stage is determined by trapped electrons and untrapped ions. If $w > \delta_e$, we have $\gamma_{NL}(w) \cong \gamma_L a(\delta_e/w)$, so that under the condition $a \ll 1$ the growth rate γ_{NL} decreases rapidly from $\gamma(0) \sim \gamma_L$ to the small value $\sim \gamma_L a$. For modes with $a \lesssim 1$, which are slightly unstable in the linear stage, in contrast, γ_{NL} changes from a small value $\gamma(0) \sim \gamma_L(1-a) \ll \gamma_L$ and $\gamma_{NL} \sim \gamma_L(\delta_e/w)$ as the instability sets in.

A very important point is that in the nonlinear stage the instability "forgets" its linear history. Under these conditions, the onset of the instability may be hard. Let us consider modes with $a>1$ below the linear stability threshold (Fig. 1). Perturbations of finite amplitude $w_0 \sim \delta_e$ grow rapidly in the nonlinear stage, $\gamma_{NL}(w_0) \sim \gamma_L a(\delta_e/w_0)$, although they are stable. A point of fundamental importance here is that the spontaneous reconnection (the development of the tearing mode) has metastable properties in configurations with magnetic shear [such as configuration (1)].

The interaction of the nonlinear mode with the untrapped ions is limited by the width of the interaction region, in which we have $E_\parallel \neq 0$: $w < |x| < \delta_\phi$. If w becomes comparable to δ_ϕ, essentially no untrapped ions will remain in the interaction region,

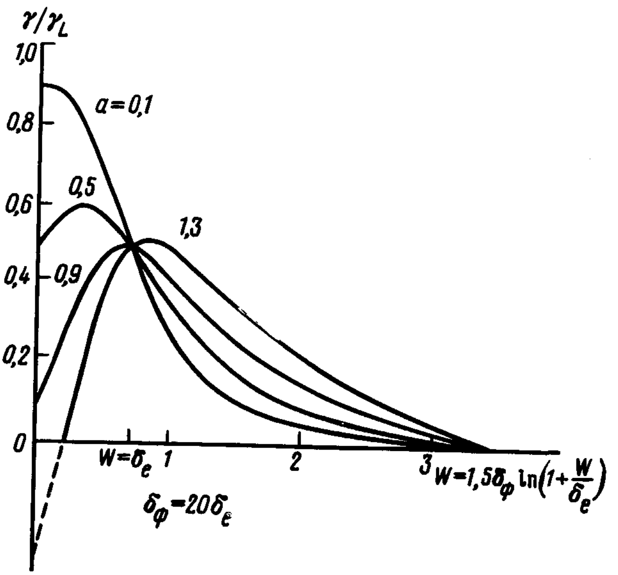

FIG. 1. The nonlinear instability growth rate versus the thickness of a magnetic island. The parameter a is a measure of the extent to which the stability threshold in the linear approximation is exceeded.

and the instability will vanish. This conclusion is confirmed by an exact numerical solution of Eq. (14), which yields $\gamma_{NL} \sim F_2(w) = 0$ at

$$w = w_{\rm sat} = 1{,}5\,\delta_\phi\,. \tag{15}$$

In the case under consideration here, with $B_y/B_0 \gg 1$, with $\rho_i \ll \delta_i$, the quantity $\delta_\phi = \sqrt{\rho_i \delta_i}$ is small in comparison with δ_i, so that result (15) confirms our assumption that the instability evolves in the interval $0 < w < w_{\rm sat} \ll \delta_i$.

In the nonlinear stage, the instability is complex: No single particle species determines the dynamics of the instability, and the electrostatic component of the field must be taken into account along with the induction component. The islands grow in size [see (14)] in a roughly linear way over time and ultimately reach the size $w \sim \delta_\phi$ (large in comparison with that in the previous calculations). The maximum amplitude of the magnetic perturbations here is $b^*_{\rm sat} = B_1/|B| \sim (dk_\parallel/dx)(w^2/2) \sim \omega_e/\omega_{Bi}$. An estimate of this quantity for the conditions in a tokamak shows that the fluctuations can reach the rather high level $\sim 10^{-4}$, so that transverse transport should be affected. Furthermore, in many systems, even systems that are stable in the linear stage, magnetic fluctuations with amplitudes above a threshold value $b^0 > b^*_{\rm thr} = b^*_{\rm sat}(m_e L_S/m_i L)$ begin to grow algebraically over time. This circumstance may be pertinent to the spontaneous "metastable" nature of reconnection in many phenomena in plasmas in the laboratory and in space.

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