Multiplicity distribution in deep inelastic production of neutrinos

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The observable KNO multiplicity distribution of secondary hadrons in deep inelastic production of neutrinos can be described by a model with a plateau-shaped inclusive spectrum and weak rapidity correlations between the primary particles if the law of conservation of momentum and of resonance decay is taken into account in this model.

The multiplicity distribution in deep inelastic neutrino production has recently been measured many times.[1,2] These measurements were carried out at energies $W \leqslant 15$ GeV ir the c.m. system where the production of a single u-quark jet is the dominant channel and the contribution from the $d,s,...$quark jets and the multiple-jet processes is evidently small.

A striking property of these multiplicity distributions, although they are much narrower than those in pp scattering, is their approximate KNO scaling.[3] In most models considered in the literature, it is assumed that a multiplicity distribution in single-jet processes is a Poisson-like distribution and that the KNO scaling observed in pp interactions arises due to the multiple-jet nature of these interactions.[4,5]

Let us assume that the dynamic correlations between the primary[1)] particles in the quark jet are small, as in the case of Poisson multiplicity distribution. We will, however, take into account the kinematic correlations associated with the momentum-conservation law. In other words, we assume that the inclusive N-particle spectrum of primary particles is given by

$$F_N(x_1, \dots, x_N) = [F_1(x_1) \dots F_1(x_N)]\,\theta\,(1 - x_1 - \dots - x_N). \qquad (1)$$

Here x_i is the Feynman variable of the i-th primary particle. The θ function in (1) takes the momentum-conservation law into account; without this function we would have a Poisson multiplicity distribution.

To transform an inclusive N-particle spectrum F_N to a multiplicity distribution w_N, we can use the relation

$$W(z) \equiv \sum_{k=0}^{\infty} z^k w_k = \sum_{k=0}^{\infty} (1/k!)(z-1)^k f_k\,, \qquad (2)$$

where

$$f_k \equiv \langle n(n-1)\dots(n-k+1)\rangle \int \prod_{i=1}^{k} dx_i F_k(x_1, \dots, x_k). \qquad (3)$$

Substituting (1) and (3) into (2) and calculating the integrals, we find

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$$W(z) = (1/2\pi i) \int d\xi / (-\xi) \exp[-\xi + (z-1)\varphi(\xi)]. \qquad (4)$$

The integration path in (4) is situated in the left half-plane parallel to the imaginary axis, and

$$\varphi(\xi) = \int_0^\infty \exp(\xi x) F_1(x). \qquad (5)$$

A very simple, physically natural model for $F_1(x)$ is

$$F_1(x) = (1/x) C(x, x_0), \qquad (6)$$

where $C(x,x_0) \approx 1$ for $x > x_0$ and $C = 0(x)$ for $x \ll x_0(x_0 = s_0/s$, where s_0 is a constant, and s is a standard Mandel'shtam variable). Substituting (5) and (6) into (4), we finally find[2]

$$W(z) = [(s/s_0)^{z-1}/\Gamma(z)](1 + O(s_0/s). \qquad (7)$$

From (7)[3] and (2) we can easily reconstruct, at least numerically, the multiplicity distribution of the primary particles, w_k.

We will now allow for the fact that we found a multiplicity distribution of the primary hadrons, a fraction of which are resonances. The subsequent decay of these resonances undoubtedly distorts the multiplicity distribution. Furthermore, a multiplicity distribution of charged particles has been observed experimentally, whereas w_k is a total-particle distribution.

We easily see that if w_k^{I} is the probability for the production of k primary particles and w_n^{II} is the probability for the production of n secondary charged particles, we will have

$$w_n^{\mathrm{II}} = \sum_{k=0}^{\infty} M(n;k) w_k^{\mathrm{I}}, \qquad (8)$$

where $M(n;k)$ is the probability that n secondary charged hadrons will form from k primary particles after the decay of all resonances.

The procedure for calculating $M(n;k)$ is shown in Fig. 1, using as an example the reaction

$$\nu + p \to \mu^- + X^{++}. \qquad (9)$$

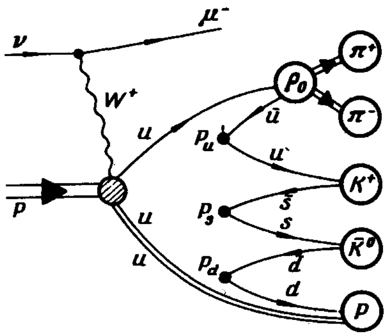

FIG. 1.

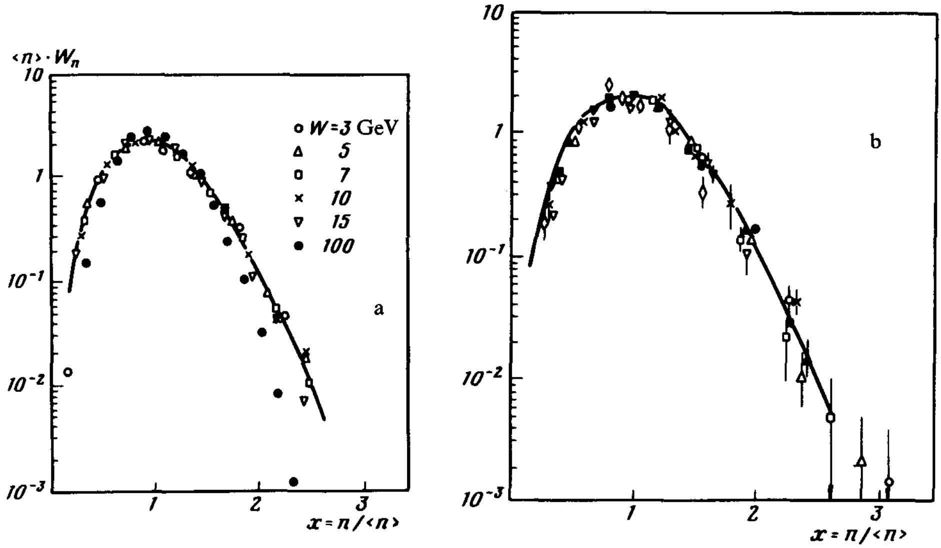

FIG. 2. The multiplicity distribution of charged hadrons in the decay $\nu + p \to \mu^- + X^{++}$ at various energies W. a—Theory; b—experiment. The experimental data were taken from Refs. 1 and 2 (the dark and light points, respectively). The curve in Fig. 2a was traced by hand and redrawn in Fig. 2b to compare theory with experiment.

A virtual W^+ boson knocks out a d quark from a proton, transforming it into a u quark and leaving the uu diquark in the proton. Furthermore, a certain number of additional $q\bar{q}$ pairs are produced from the vacuum. These pairs form the primary hadrons after recombining with the original quark and diquark and with each other.[4] The probabilities for the production of u,d, and s quarks in this case are assumed to be (see Ref. 6)

$$P_u = P_d = 0{,}4; \qquad P_s = 0{,}2. \tag{10}$$

The spin of the produced primary hadron is found from the quark spin combinatorial analysis and the hadron species for a given spin is found from the quark structure of the hadron. The unstable primary particles decay. The probabilities for various decay channels are taken from the experiment of Ref. 8. Under these conditions $M(n;k)$ can be determined with a reasonable accuracy by Monte Carlo calculations.

Knowing $M(n;k)$ and w_k^{I}, we can easily find the charged-particle multiplicity distribution for decay (9), shown in Fig. 2a for various energies. We see that the multiplicity distributions found for the energy 3 GeV$\leqslant W \leqslant$15 GeV are in satisfactory agreement with the approximate KNO scaling and, as is evident from Fig. 2b, are in good agreement with the experiment.

We should point out that in this context the approximate KNO scaling is perceived as an adequately random phenomenon. Furthermore, this scaling breaks down at reasonably high energies, as can be seen in Fig. 2a.

I wish to thank O. V. Kancheli for valuable comments.

[1)]By primary particles we mean, following Ref. 6, those particles which are produced directly in the interaction. The observed secondary particles are stable particles and decay products of unstable primary particles.

[2)]The explicit form of the function $C(x,x_0)$ in first approximation in s_0/s determines only the value of the constant s_0.

[3)]The limiting form of (7) for the generating multiplicity distribution function was established previously[7] in terms of a two-dimensional jet model.

[4)]Note that the number of primary hadrons and hence of additional $q\bar{q}$ pairs is fixed.

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