Higgs mechanism and gravitation

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A Lagrangian of the Higgs type arises in a natural way when the gravitational interactions associated with a scalar field are taken into account.

1. The Klein-Gordon equation is constructed on the basis of the principle of Lorentz covariance. The Higgs Lagrangian, on the other hand, is introduced in an *ad hoc* way and is justified only by its success (the Higgs mechanism). Our purpose in the present letter is to show that a Lagrangian of the Higgs type arises in a natural way (without any appeal to additional hypotheses) when gravitational interactions associated with a scalar field are taken into account.

2. We consider a scalar field φ which obeys the equation

$$\Box\varphi + m^2\varphi - (R/6)\varphi = 0; \tag{1}$$

we choose the signature and curvature in accordance with Ref. 1, so that m^2 has the "correct" sign (not the "tachyon" sign). To determine the curvature, we need Einstein's equations:

$$R_{ik} - {}^1/_2\, g_{ik} R = kT_{ik}, \tag{2}$$

where T_{ik} is the energy-momentum tensor of *all* of the fields present in the given model. This circumstance is exceedingly important: According to the meaning of (2), the tensor T_{ik} must describe *all* the matter present in the vicinity of the given point. The corresponding curvature is clearly many orders of magnitude greater than the average cosmological curvature, which is sometimes substituted into (1) as R (while R has the meaning of a local quantity). We thus set $T_{ik} = T_{ik}(\varphi) + T^{(0)}_{ik}$ in (2), where the first term corresponds to the field φ, and the second to all the other matter. We then have $R = -kT(\varphi) - K$, where $T(\varphi) = \text{trace}[T_{ik}(\varphi)]$, and $K/k = \text{trace}(T^{(0)}_{ik})$. Calculating $T_{ik}(\varphi)$ by a variation in the metric of the Lagrangian corresponding to (1), we find $T(\varphi) = 2m^2\varphi^*\varphi$, so that (1) becomes

$$\Box\varphi + (m^2 - K/6)\varphi + (km^2/3)(\varphi^*\varphi)\varphi = 0. \quad (1')$$

The corresponding Lagrangian is of the Higgs form,

$$\mathcal{L} = \sqrt{-g}\,[\varphi^*_{,n}\varphi^{,n} - (m^2 - K/6)\varphi^*\varphi - (km^2/6)(\varphi^*\varphi)^2], \quad (3)$$

and under the condition $K > 6m^2$ we find a spontaneous symmetry breaking: The Hamiltonian has a minimum at $\varphi^*_0\varphi_0 = (3/k)[(K/6m^2) - 1]$, where it has the value $\mathscr{H}_0 = -(km^2/6)(\varphi^*_0\varphi_0)^2$.

3. We now introduce a massless vector field A_i. Adding the Lagrangian of the field A_i, which is $-1/4(A_{i,k} - A_{k,i})^2$, to the Lagrangian corresponding to (1), and using the replacement $\varphi_{,k} \to \varphi_{,k} - ieA_k\varphi \equiv D_k\varphi$, we find that the trace of $T_{ik}(\varphi, A)$ is $2m^2\varphi^*\varphi$, so that instead of (3) we find

$$\mathcal{L} = \sqrt{-g}\,[D^n\varphi(D_n\varphi)^* - (m^2 - K/6)\varphi^*\varphi$$
$$- (km^2/6)(\varphi^*\varphi)^2 - {}^1\!/_4(A_{i,k} - A_{k,i})^2]. \quad (4)$$

Shifting the field φ by an amount φ_0 (Section 2), we find, working in the usual way, that the scalar and vector fields acquire masses $m_s^2 = (2/3)(K - 6m^2)$ and $m_v^2 = (3e^2/2k)(m_s/m)^2$, respectively.

4. Accordingly, in working exclusively from Eq. (1) (treated here as a standard equation for the scalar field), we have obtained the Higgs mechanism as a consequence of the gravitational self-effect of the scalar field φ and of the presence of a nonvanishing background curvature corresponding to all the other fields. The parameters K and m of course remain undetermined. All that we can say about the parameter K, which corresponds to the background curvature, is that it may be exceedingly large. According to the direct meaning of (2), K should be taken at a point (and a divergence may evidently occur), but if we restrict the region to a diameter on the order of the Planck diameter, we would expect a value in the Planck range, 10^{66} cm^{-2}, for K. Incidentally, it might be better to average (2) over the region characteristic of the particular interactions under consideration (e.g., over a Fermi region with a diameter of 10^{-17} cm), but at this point we can say nothing definite regarding the average curvature in these regions. It is clear, however, that in this case K may depend on the state of the background matter. Expressing K in terms of the pressure and density of this matter, $K = \kappa(\epsilon - 3p)$, we see that an increase in the pressure may cause a violation of the condition $K > 6m^2$, which would lead to a restoration of the symmetry.

5. Introducing a spinor field ψ with a mass μ, which interacts only with A_i (and with the metric)—not with φ—so that the trace of T_{ik} acquires an increment $\mu\bar{\psi}\psi - e\bar{\psi}\gamma^k\psi A_k$, we find, by shifting the field φ, that the charge and mass of the field ψ change in accordance with $e \rightarrow e(1 - k\varphi_0^2/6$, $\mu \rightarrow \mu(1 + k\varphi_0^2/6)$, and we find a Yukawa coupling with a scalar field with a coupling constant $k\mu\varphi_0/6$. If the Yukawa coupling with the field φ is introduced at the outset, this quantity would of course represent a correction to the coupling constant.

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[1]L. D. Landau and E. M. Lifshitz, Teoriya polya, Nauka, Moscow, 1973 (The Classical Theory of Fields, Pergamon, New York, 1976).

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