Mechanism for proton precipitation triggered by a VLF wave injected into the magnetosphere

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A mechanism for the interaction of fast protons with a VLF wave in the inhomogeneous magnetospheric plasma is proposed to explain the observed features of stimulated precipitation {R. A. Kovrazhkin *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 193 (1984) [JETP Lett. **39**, 228 (1984)]}.

The effects of the resonant interaction of VLF signals radiated by ground-based transmitters with fast electrons of the magnetospheric plasma have been the subject of active experimental and theoretical research over the past decade (see Ref. 2 and the bibliography there). The analogous interaction for protons, in contrast, has no significant effect on either the wave or the particle distribution function, since (first) the resonant conditions,

$$\omega - k_{\parallel} v_{\parallel} = n\Omega_i, \quad (n = 0, \pm 1, \ldots) \tag{1}$$

(the notation is standard), require overly high values of $v_{\parallel}$ for the typical properties of the magnetosphere and (second) the interaction in a given resonance is weaker by a factor m_e/m_i than that for electrons. The discovery of a stimulated precipitation of protons[1] on the Oreol-3 satellite was accordingly a somewhat unexpected experimental result. The preliminary interpretation offered for this result by Kovrazhkin *et al.* had the precipitation resulting from an interaction of protons with a VLF wave at a Čerenkov resonance [$n = 0$ in Eq. (1)]. This interpretation is unconvincing, however, for the reasons specified above, and we need to search for some other, more effective, mechanism for the precipitation of protons in a VLF wave. For this purpose we have derived a theory for the interaction of protons with a VLF wave at higher-order cyclotron resonances in the inhomogeneous magnetospheric plasma. Because of the inhomogeneity of the medium, protons which are moving along a line of force of the geomagnetic field B pass through a large number of resonant regions and accordingly undergo a pitch-angle diffusion and are precipitated from the magnetosphere.

As we show below [see condition (7)], the energy of the protons interacting with the wave decreases substantially after the wave goes into an electrostatic regime, in which its refractive index increases. Choosing a potential $\Phi = \Phi_0 \sin(\int k_{\parallel} ds + k_{\perp} r_{\perp} - \omega t)$ for a wave propagating in the meridional plane, we write the Hamiltonian of a particle in the external magnetic field $\mathbf{B}$ and in the wave field $\vec{\nabla}\Phi$ as follows:

$$H = \frac{v_{\parallel}^2}{2} + I\Omega(s) + (q\Phi_0/m)\sin\psi; \quad \psi = \int^{s} k_{\parallel} ds' + k_{\perp}\sqrt{2I/\Omega}\sin\varphi - \omega t. \tag{2}$$

Here the canonical variables are $(v_{\|}, s)$, the longitudinal velocity $v_{\|}$ and the coordinate s along a geomagnetic line of force, and (I, φ), the transverse adiabatic invariant $I = v_{\perp}^2/2\Omega$ and the gyrophase φ of the particle; here $\Omega = qB/mc$ is the proton gyrofrequency. Introducing

$$\epsilon = q\Phi_0/m; \qquad \mu = k_{\perp}\sqrt{2I/\Omega}; \qquad \xi_n = \int^{s} k_{\|}ds' + n\varphi - \omega t, \tag{3}$$

we can rewrite Hamiltonian (2) as

$$H = \frac{v_{\|}^2}{2} + I\Omega(s) + \epsilon \sum_{n=-\infty}^{\infty} J_n(\mu)\sin\xi_n, \tag{4}$$

where $J_n(\mu)$ is the Bessel function of index n. In the approximation of isolated resonances, we need retain only the n-th term in (4). Since the variables φ and t appear in the Hamiltonian in the combination $n\varphi - \omega t$ in this case, the quantity

$$C_n = nH - I\omega \tag{5}$$

is an integral of motion, and the equations of motion are

$$\begin{gathered} \dot{s} = v_{\|}; \qquad \dot{v}_{\|} = -I d\Omega/ds - \epsilon k_{\|} J_n(\mu)\cos\xi_n; \\ \dot{\varphi} = \Omega(s) + [\epsilon k_{\perp} J_n'(\mu)/\mu\Omega]\sin\xi_n; \qquad \dot{I}_n = -\epsilon n J_n(\mu)\cos\xi_n. \end{gathered} \tag{6}$$

It follows from (6) that an effective interaction of the particle with the wave occurs only if $\mu \equiv k_{\perp}v_{\perp}/\Omega > n$ [in the opposite case, $J_n(\mu)$ is exponentially small]. Combining this condition with resonant condition (1), we find

$$k_{\|}v_{\|} + k_{\perp}v_{\perp} > \omega. \tag{7}$$

We thus see that, in contrast with the case of electrons, the condition for an effective interaction of protons with the wave is not a condition of the form in (1) (which can always be satisfied by choosing an appropriate value of n) but inequality (7), under which the variable ψ in (2) has a stationary-phase point on the cyclotron circle.

To evaluate the change in I as the region of the cyclotron resonance is crossed, we write an expression for $\ddot{\xi}_n$, which follows from (6):

$$\ddot{\xi}_n = -\epsilon k_{\|}^2 J_n(\mu)\cos\xi_n - a, \tag{8}$$

where

$$a = k_{\|}\left(I\frac{d\Omega}{ds} + \frac{1}{2}\frac{dv_{Rn}^2}{ds}\right); \qquad v_{Rn} = \frac{\omega - n\Omega}{k_{\|}}.$$

The parameter a is a measure of the inhomogeneity for the n-th resonance, and the ratio $\beta = \epsilon k_{\|}^2 J_n/a$ is a measure of the relative importance of the nonlinearity and the inhomogeneity in the resonant interaction of waves and particles. Under the condition $|\beta| \ll 1$ (which held under the experimental conditions of Ref. 1), the parameter a determines both the time of the resonant interaction, $|a|^{-1/2}$, and the time it takes the particle to traverse the distance between two adjacent resonances,

$$\tau = \Omega / |a| . \qquad (9)$$

Clearly, the resonances are isolated if $|a|^{1/2} \ll \Omega$. In the vicinity of a resonance, the phase ξ_n is, according to (8),

$$\xi_n = \xi_{nR} - at^2/2 , \qquad (10)$$

where we have used the condition $\dot{\xi}_n = 0$ at the point of the resonance. Substituting (10) into (6), and integrating the equation for I, we find

$$\Delta I = -(2\pi / |a|)^{1/2} \epsilon n J_n(\mu) \cos[\xi_{nR} - \mathrm{sign}\, a\, (\pi/4)] . \qquad (11)$$

The corresponding change in the kinetic energy of the particle, $W = v^2/2$, is related to ΔI by $\Delta W = (\omega/n)\Delta I$, which follows from (5). Since the particle moves freely in phase space away from a resonance, the values of the phases ξ_{nR} in adjacent resonances are uncorrelated. We thus have a diffusion of particles in phase space, which develops with a scale time (9) and which is described by the diffusion coefficients [see (9) and (11) and the comment following the expression]

$$D_{II} \equiv \frac{<\Delta I^2>}{2\tau} = \frac{\pi \epsilon^2 \tilde{n}^2}{4\mu\Omega} ; \quad D_{IW} = D_{WI} = \frac{\omega}{\tilde{n}} D_{II} ; \quad D_{WW} = \frac{\omega^2}{\tilde{n}^2} D_{II} , \qquad (12)$$

where

$$\tilde{n} = [\omega - k_{\parallel}(s) v_{\parallel}] / \Omega(s) ; \quad v_{\parallel} = \sqrt{2[W - I\Omega(s)]} .$$

Knowing the diffusion coefficients, we can use the standard procedure to calculate the change in the number of particles in the loss cone, which determines the flux density of particles precipitated from the magnetosphere. The result is

$$j = 2\pi\Omega_0 \int ds' \int_{W_{min}}^{\infty} dW \left\{ \left(1 - \frac{\omega}{\tilde{n}\Omega_0}\right) \frac{D_{II}}{v_{\parallel}} \left[\frac{\partial F(I, W)}{\partial I} + \frac{\omega}{\tilde{n}} \frac{\partial F(I, W)}{\partial W} \right] \right\}_{I = W/\Omega_0} .$$

Here $F(I, W; s', t)$ is the particle distribution function, $\tilde{\Omega}_0$ is the cyclotron frequency at the level of the ionosphere, the integration over s' is carried out over that part of the line of force on which the wave is electrostatic, and $2W_{min} = (\omega/k_{max})^2$.

Because of the uncertainty regarding the proton distribution function near the boundary of the loss cone, the results of this theory should be compared primarily with the kinematic features of the observed proton precipitation.[1] With $\omega \sim 10^5$ s^{-1} and $k_{max} \sim 2 \times 10^{-4}$ cm^{-1}, condition (7) gives us a lower limit on the energy, which simultaneously corresponds to the maximum differential flux density of precipitated protons: $E \sim 100$ keV. The wave goes into the electrostatic regime in the hemisphere opposite the injection hemisphere,[3] and the wave moves to lower L shells. Since the protons move along lines of force, the precipitation maximum should be observed at $L < L_{inj}$, and the delay between the precipitation time and the time at which the wave is radiated should be on the order of half the sum of the bounce periods of the wave and the particle. All of these conclusions agree with the experimental results of Ref. 1. As for the absolute value of the flux density, we note that it turns out to be proportional to the number of resonances that have been crossed, $n_{\text{eff}} \sim 10^2$, so that the effectiveness of the mechanism proposed here is higher by a factor of n_{eff} than that which follows from the model of Ref. 1 for an interaction at a Čerenkov resonance.

[1]R. A. Kovrazhkin, M. M. Mogilevskiĭ, O. A. Molchanov, Yu. I. Gal'perin, N. V. Dzhordzhio, Zh. M. Boske, and A. Rem, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 193 (1984) [JETP Lett. **39**, 228 (1984)].
[2]T. F. Bell, J. Geophys. Res. **A89**, 905 (1984).
[3]Yu. K. Alekhin and D. R. Shklyar, Geomagn. Aeronom. **20**, 501 (1980).

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