Ponderomotive effects in the near zone of an antenna in a magnetized plasma

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The ponderomotive force created by the electromagnetic field is incorporated in an analysis of the structure of the near zone of an antenna in a magnetized plasma. If the current amplitude in a loop antenna exceeds a certain threshold I_0, the structure in the near zone changes qualitatively. The threshold current I_0 is close to values which have been achieved experimentally.

The nonlinear effects which occur near a transmitting antenna are an important consideration in the analysis of experiments in plasmas in the laboratory[1,2] and in space.[3] In a certain intensity interval the most important nonlinear effects are those which result from the change caused in the plasma density by the ponderomotive force of the electromagnetic field. These effects are the subject of the present letter.

The ponderomotive effects should be strongest in the near zone of the antenna, i.e., within distances of the antenna small in comparison with the length of the radiated wave. Under these conditions, the displacement currents can be ignored, so that the equation for the magnetic vector potential **A** becomes $\Delta \mathbf{A} = -(4\pi/c)\mathbf{j}$, $\nabla \mathbf{A} = 0$. The electric field is determined by the equations

$$\mathbf{E} = (i\omega/c)\mathbf{A} - \nabla\psi, \quad \mathrm{div}\,\mathbf{D} = 4\pi\rho, \tag{1}$$

from which we find

$$\mathrm{div}\,(\hat{\epsilon}\nabla\psi) = (i\omega/c)\mathrm{div}\,(\hat{\epsilon}\mathbf{A}) - 4\pi\rho, \tag{2}$$

where $\hat{\epsilon} = (\epsilon_{ik})$ is the dielectric tensor of the plasma, which depends on the plasma density N and the steady-state magnetic field $\mathbf{B}_0$, and $\mathbf{j}$ and ρ are the current density and the charge density in the antenna. We assume that the plasma is collisionless and that the condition $8\pi NT \ll b_0^2$ holds (for simplicity, we assumed $T_\parallel = T_\perp$). Under these conditions, the distortion of the external magnetic field by ponderomotive effects can be ignored, and the nonvanishing components of $\hat{\epsilon}$ are $\epsilon_{xx} = \epsilon_{yy} = \epsilon$, $\epsilon_{xy} = -\epsilon_{yx} = -ig$, $\epsilon_{zz} = \eta$. These components are described by the known expressions for a "cold" plasma. We restrict the discussion below to the axisymmetric case, in which the antenna axis is directed along $\mathbf{B}_0$ (the z axis). Introducing the cylindrical coordinates r, φ, z, and assuming that the unknown quantities do not depend on φ, we find the following expression from the MHD equations, supplemented with the ponderomotive force:

$$N = N_0 \exp\{(32\pi NT)^{-1}[(\epsilon - 1)(|E_r|^2 + |E_\varphi|^2) + (\eta - 1)|E_z|^2 + ig(E_\varphi^* E_r - \mathrm{c.c.})]\}. \tag{3}$$

We note that $\epsilon - 1$, $\eta - 1$, and g are all proportional to N.

As an application of Eqs. (1)–(3) we consider a magnetic antenna carrying a circular current (of radius a) which is perpendicular to $\mathbf{B}_0$. For this case Eq. (2) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\epsilon\frac{\partial\psi}{\partial r}\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial\psi}{\partial z}\right)=\frac{\omega}{cr}\frac{\partial}{\partial r}(rgA_\varphi), \tag{4}$$

where $\mathbf{A}$ is given by the well-known expressions,[4] and we have $A_r = A_z = 0$. In particular, for $r^2 + z^2 \gg a^2$ we have

$$A_\varphi = Mr(r^2+z^2)^{-3/2} \qquad M=\pi a^2 I/c, \tag{5}$$

where M is the magnetic moment, and I is the current.

The primary distinctive feature of Eq. (4) is that the signs of the functions $\epsilon(N, B_0, \omega)$ and $\eta(N, B_0, \omega)$ can differ, depending on the values of the arguments. For example, in the whistler frequency range, i.e., under the conditions $(\omega_{ce}\omega_{ci})^{1/2} \ll \omega < \omega_{ce} \ll \omega_{pe}(N)$, we have $\epsilon > 0$ and $\eta < 0$, as we assume below (for definiteness).

We first find the solution of the linearized version of Eq. (4), (i.e., for $N = N_0$) in case (5). In this case we can seek a solution of Eq. (4) in the self-similar form $\psi = |z|^\nu f(r^2/z^2)$. We find $\nu = -1$, and for $f(w)$ we find an inhomogeneous hypergeometric equation. From the solution of this equation, which satisfies the regularity conditions at $r = 0$, we find

$$\psi = -\alpha(1+\gamma^2)^{-1}R^{-1}, \quad R^2 = r^2 + z^2, \tag{6}$$

$$\alpha = \omega M g_0/c\,|\eta_0|, \quad \gamma^2 = \epsilon_0/|\eta_0| \tag{7}$$

[$\epsilon_0 = \epsilon(N_0)$, etc.]. This solution is extremely useful for estimates and also for analyzing more-complicated problems, in particular, nonlinear problems. For example, substituting into (3) the expression found for $\mathbf{E}$ from (1), (5), and (6), we find an estimate of $\Delta N = N - N_0$ (for whistlers):

$$\Delta N/N_0 \approx -D^2(1+\gamma^2)^{-2}(a/R)^4 \qquad (\Delta N \ll N_0), \tag{8}$$

$$D = (\pi/2c^2)e\,|g_0/\eta_0|(2m_e T)^{-1/2} I. \tag{9}$$

We can now find a self-consistent nonlinear solution in the case $\gamma^2 \approx (\omega/\omega_{ce})^2 \ll 1$ and $r \ll z$; we also take into account the dimensions of the antenna. Here we may assume $\epsilon \gg 1$, $|\eta| \gg 1$, i.e., that ϵ and η are proportional to N. Noting that we have $E_r = 0(r)$ and $\partial E_z/\partial r = 0(r)$ near the z axis,

$$N = N_0 \exp[-\mu^2(z)] + O(r^2), \qquad \mu = -(\omega_{pe}/\omega)(32\pi N_0 T)^{-1/2}E_z, \tag{10}$$

and ignoring terms on the order of γ^2, we find an ordinary differential equation for $\mu(z)$. Solving this equation under the condition $\mu \to 0 (z \to \infty)$, we find

$$\mu - (2/3)\mu^3 = 2D[1 - z(z^2+a^2)^{-1/2}]. \tag{11}$$

At $z \gg a$ we find from (11) $\mu \approx D(a/z)^2$, which is equivalent to (6) under the condition $r \ll z$. Examining (11), we easily find that at

$$D < D_0 = (18)^{-1/2} \approx 0{,}24 \tag{12}$$

the function $\mu(z)$ falls off continuously from $\mu_{max} \approx 2D + (16/3)D^3$ (at $z = 0$) to zero (at $z = \infty$). At $D > D_0$ there exists a point z_0,

$$z_0 = a(D - D_0)[(2D - D_0)\ D_0]^{-1/2}, \tag{13}$$

at which we have $d\mu/dz = \infty$ as $z \to z_0$. Here $\mu(z_0 + 0) = \mu_+ = (1/2)^{1/2}$, and $\mu(z_0 - 0) = \mu_- = -\sqrt{2}$ (Fig. 1).

Summarizing, we can say that the nonlinear effects described by Eq. (11) set in after a threshold is reached. For $D \leqslant D_0$, the field varies continuously from $z = \infty$ to $z = 0$. Here we have

$$\mu_{max} = \mu(0) \leqslant \mu_+ \approx 0{,}71; \quad 1 > N/N_0 > N(0)/N_0 \approx 0{,}61.$$

For $D > D_0$ the structure of the near zone of the magnetic antenna changes qualitatively: There exists a point $z_0 > 0$ at which the field changes abruptly according to (11), and we have $dE/dz \to \infty$ as $z \to z_0 + 0$. The events that occur close to z_0 are not described by Eqs. (3) and (4) and will be the subject of a future study. We do not rule out the possibility that the plasma undergoes a local heating near z_0.

The current in the antenna which corresponds to $D > D_0$ can be found from (9) for the condition $(\omega/\omega_{ce})^2 \ll 1$:

$$I \geqslant I_0 \approx 0{,}047\,(\omega_{ce}/\omega)\,T^{1/2}\ \text{(amperes)}. \tag{14}$$

Here T is expressed in degrees. For the upper ionosphere [$H = (1\text{–}3) \times 10^3$ km] with $\omega = 0.03\omega_{ce}$ we have $I_0 \approx 100a$, which is approximately the current in the experiments

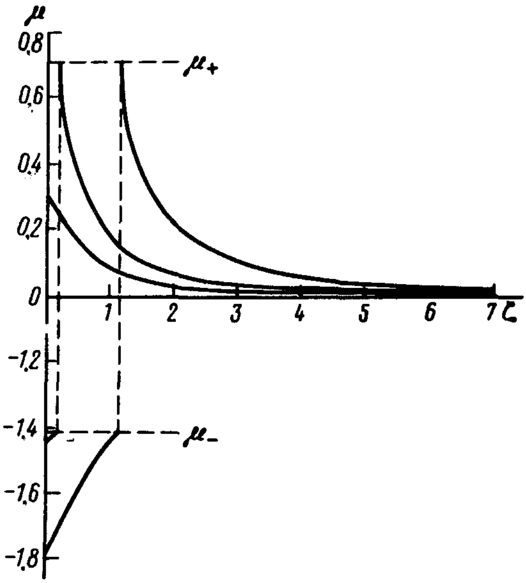

FIG. 1. Solutions of Eq. (11) satisfying the condition $\mu(z) \to 0$ as $z \to \infty$ for various values of D: 1—0.15; 2—0.30; 3—1.

of Ref. 3. In the magnetosphere, at distances $(3-4)R_E$, in the equatorial region, experiments are usually carried out at $\omega \sim (0.3-0.5)\omega_{ce}$, and we have $T \sim 10^4$. We thus find $I_0 \sim 10a$. We find a value for I_0 on the same order of magnitude from (14) for the laboratory experiments of Refs. 1 and 2. Here the condition $\lambda > 2\pi a$ holds, where λ is the wavelength.

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