Metastable dissipative structures in media with thermal diffusion

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The existence of metastable dissipative structures, i.e., spatially periodic formations which are unstable but long-lived, is predicted for the specific case of the dynamics of a gaseous mixture of light and heavy particles which are interacting with external radiation. The observation of such structures would substantially improve the understanding of self-organization in nonlinear distributed systems.

It has been solidly established that processes that give rise to a spatial and temporal ordering can occur in media which exhibit a nonlinear interaction and which are "open" in terms of energy (or mass) transfer. Various mechanisms which lead to the formation of stable, inhomogeneous stationary states—dissipative structures—are currently the subject of active research.[1–3] The evolutionary processes that lead to the formation of dissipative structures can in general be exceedingly complicated. In the present letter we show that metastable dissipative structures, i.e., formations which are unstable but which persist for a rather long time (long enough for experimental observation, for example), may arise in the course of such processes.

The most important distinctive feature of metastable dissipative structures is that, while being unstable stationary states, they nevertheless unavoidably form from various initial conditions, and (in sufficiently large systems) they depend only slightly on the boundary conditions. The existence of metastable dissipative structures does not contradict the principles of statistical physics. To the best of our knowledge, however, no examples of physical systems, in which metastable dissipative structures might

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arise, have been discussed in the literature. A study of such structures is of considerable importance for reaching an understanding of the evolutionary properties of nonlinear distributed systems.

Let us examine the heating of a gaseous mixture by external radiation. We assume that the gas consists of two components (B_l and B_h) which do not interact with each other and that the radiation is selectively absorbed by the lighter component, B_l, which has a density n in the range $0<n<1$. We assume that the gas is in a long (along the X axis), thin cell (of thickness h) and that radiation with a uniform intensity distribution $I(x)=I=$ const is incident on the gas (perpendicular to the X axis). If the thermophysical characteristics of the components B_l and B_h differ only insignificantly, the heating of the gas can be described by a system of two equations, the heat-conduction equation and the diffusion equation[4]:

$$\frac{1}{a}\frac{\partial T'}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{1}{\kappa h}\left[\beta h n I-\eta(T-T_e)\right], \tag{1}$$

$$\frac{\partial n}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial n}{\partial x}\right)+\frac{\partial}{\partial x}\left(D_T\frac{\partial T}{\partial x}\right),\quad D_T=\alpha D\frac{n(1-n)}{T}, \tag{2}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0,L}=0,\quad \left.\frac{\partial n}{\partial x}\right|_{x=0,L}=0,\quad n|_{t=0}=n_0. \tag{3}$$

Here a, κ, and D are the thermal diffusivity, the thermal conductivity, and the diffusion coefficient (all assumed to be constant); β is the absorption coefficient of B_l; T_e is the temperature of the external medium; and η is the heat-transfer constant. We know that thermal diffusion, embodied in the last term in (2), is an important transport mechanism in gases. The thermal diffusion flux for the lighter component, B_l, is directed toward the hotter part of the medium, so that the thermal diffusion constant satisfies $\alpha<0$. At this point, we set $a=D$.

It follows from (1)–(3) that in a homogeneous stationary state we would have

$$n=n_0,\ T=T_0=T_e(1+I/I_0),\qquad I_0=\eta T_e/\beta h n_0. \tag{4}$$

Setting $T=T_0+\theta$ and $n=n_0+\nu[\theta,\ \nu\sim\exp(pt+iqx)]$ and linearizing (1)–(3) with respect to the small increments (θ,ν), we find the dispersion relation

$$p_{1,2}=t_0^{-1}\left[-\frac{1}{2}-(qx_0)^2\pm\sqrt{\frac{1}{4}+\epsilon(qx_0)^2}\right],\quad \epsilon=\frac{|\alpha|(1-n_0)I}{I+I_0},$$

$$x_0=(\kappa h/\eta)^{1/2},\qquad t_0=\kappa h/a\eta. \tag{5}$$

It follows that at $\epsilon>1$, i.e., at $I>I_{cr}=I_0/[|\alpha|(1-n_0)-1]$, the homogeneous state is unstable: There exists an interval of wave vectors q, $q^2<q_0^2=(\epsilon-1)/x_0^2$, in which the condition $p_1(q)>0$ holds. We note that we have $p_1(0)=p_1(q_0)=0$, and at the point of the maximum we have

$$p_{max}=p(q_m)=(\epsilon-1)^2/4\epsilon t_0,\quad q_m^2=(\epsilon^2-1)/4\epsilon x_0^2. \tag{6}$$

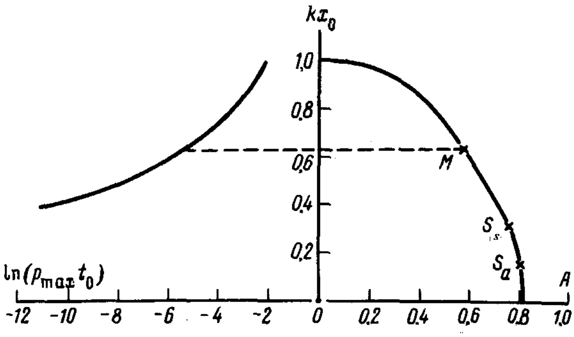

FIG. 1.

The subsequent evolution is determined by the number and stability of the inhomogeneous stationary states. Analysis shows that all the bounded stationary solutions of boundary-value problem (1)–(3) are periodic. In a region of dimension L, any solution whose period λ satisfies the condition $m\lambda = 2L$, where $m = 1, 2, \ldots$, is permissible.

It is instructive to plot the entire set of stationary solutions of (1)–(3) as the solution amplitude $A = n_{\max} - n_{\min}$ versus $k = 2\pi/\lambda$. Figure 1 shows a curve of $A(k)$ for $\alpha = -3$, $I = 5I_0$, and $n_0 = 0.2$. This curve can be used to find all the stationary solutions in a region of arbitrary dimensions L. For the particular values used for the parameters, the function $A(k)$ is single-valued and corresponds to a "soft" creation of the structures as the dimensions of the region are increased. However, there exists a region of parameter values in which $A(k)$ is ambiguous, so that the creation of the structures is "hard."

Equations (1)–(3) have been solved numerically. The results show that for a wide range of initial perturbations a distribution $n_1(x)$ which remains essentially constant, even after a doubling of the compilation time, is established in a time $t \sim 50t_0$. Curve 1 in Fig. 2 shows $n_1(x)$ for $L = 20x_0$. It is important to note that the distribution $n_1(x)$ is essentially the same as the stationary solution of system (1)–(3) with a period $\lambda = 10x_0$ (point M in Fig. 1). Region 1 in Fig. 3 shows the time evolution of n at the maxima a, b, and c in Fig. 2. The system thus evolves as if the stationary state $n = n_1(x)$ were stable, with a large attraction region (as in the case of ordinary dissipative structures).

Actually, this solution is unstable. This instability can be demonstrated by calcu-

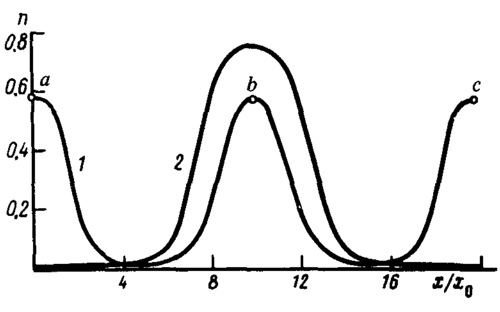

FIG. 2.

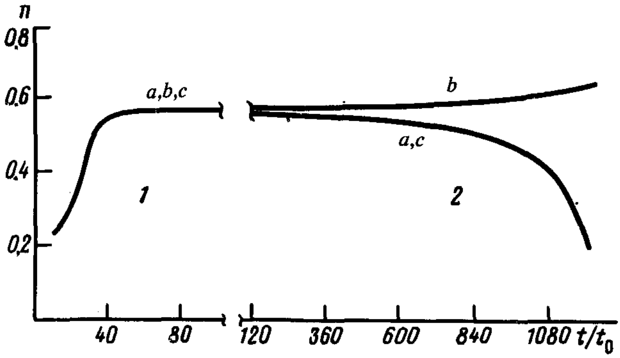

FIG. 3.

lating (by the method of Ref. 5, for example) the growth rates $p = p(q;k)$ [$q = m\pi/L$ ($m = 0, \pm 1, \pm 2,...$) is the perturbation wave vector, and $k = 2\pi/\lambda$, where λ is the period of the stationary solution in question]. In particular, it turns out that in the case in Fig. 1, in a region of dimension L, only the state S_a, a half-wave of the solution with a period $\lambda = 2L$, is stable. If, on the other hand, we require, in addition to boundary conditions (3), that the solution be symmetric with respect to the point $x = L/2$ (a cyclic condition), then the state S_s, corresponding to one wave of the stationary solution with the period $\lambda = L$, will be stable. The reason for the stability of these solutions is that all permissible wave vectors q fall in the region $p \leqslant 0$. The solution $n = n_1(x)$, on the other hand, has a period $\lambda = L/2$ and is unstable. Region 2 in Fig. 3 shows the subsequent evolution of n at points a, b, and c (in Fig. 2): the disappearance of the "extra" maxima (at points a and c). As a result, a stable inhomogeneous state is formed. Curve 2 in Fig. 2 shows the corresponding symmetric solution S_s [$n = n_2(x)$]. In Fig. 1, the point S_s corresponds to a stable symmetric solution, while the point S_a corresponds to an asymmetric solution. If we did not impose the cyclic requirement, the transition from S_s to S_a (at $L = 20x_0$) would occur in a time $t > 10^7 t_0$.

The time scale over which the solution $n = n_1(x)$ becomes unstable is determined by the growth rates for perturbations with wave vectors q near the maximum of $p(q;k)$. Figure 1 shows a curve of $p_{\max}(k) = \max_q p(q;k)$. The function $p_{\max}(k)$ decays rapidly. It follows, in particular, that the lifetime of the solution $n = n_1(x)$ ($k = 2\pi/\lambda \approx 0.628 x_0^{-1}$) is at least 30 times longer than the lifetime of the homogeneous state.

We thus see that during the intermediate stage of the evolution of $n(x,t)$ the system goes into an unstable stationary state $n_1(x)$ (a metastable dissipative structure is formed). The appearance of a metastable dissipative structure is "unavoidable" because the sharp maximum in $p(q)$ [see (5) and (6)] causes a rapid growth of perturbations with wave vectors near q_m and thus the formation of a structure with $k \approx q_m$ in the medium. Perturbations with small q subsequently begin to play an important role; they lead to a diffusive interaction of remote parts of the medium and to a breakup of the structure $n = n_1(x)$.

We wish to emphasize that trapping by an unstable stationary state may occur in many nonlinear problems and is important in research on real systems.

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