Observation of self-focusing of sound

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Self-focusing of sound in a liquid has been seen for the first time. The distributions of the wave amplitude and phase in the beam cross section have been measured at various times.

1. In this letter we report experiments in which a self-focusing of sound has been observed for the first time. We observed the contraction of a sound beam, an intensification of the sound at the axis, and a change in the curvature of the wavefront. We studied the formation of the acoustic field after the transducer was turned on. The observed results can be explained by a thermal self-effect mechanism involving a decrease in the sound velocity in a medium heated by an intense acoustic wave.

2. Self-focusing is a fundamental nonlinear wave effect. It has been studied extensively in optics.[1] In acoustics, it has been the subject of theoretical analysis[2] and calculations,[3,4] but it has not yet been observed experimentally.

The theoretical work on the self-focusing of sound takes an approach similar to that used in the description of the self-focusing of light. However, the self-focusing of sound has an important distinctive feature, which must be taken into account: When a liquid is heated by sound, there is a transfer of momentum five orders of magnitude greater than during the absorption of the same amount of luminous energy. The resulting currents[5] tend to defocus the beam.[6] The driving of these currents can be opposed, and their velocity reduced, by using a cell and a liquid with special properties. Quadratic nonlinear effects—the generation of a broad spectrum of harmonics, the formation of discontinuities,[5] etc.—also compete with the self-focusing of sound. The nonlinear decay of shock waves can of course increase the temperature gradients and amplify the effect of interest. In this case, however, the field has a complex spatial and temporal structure, and the very concept of self-focusing of sound must be altered. The self-focusing of beams which contain field discontinuities is a problem open to theoretical and experimental research.

3. We have observed self-focusing of sound in glycerin and transformer oil. The ultrasound is excited by a ceramic piezoelectric transducer $d = 3$ cm in diameter at the frequency $f = 1$ MHz. The sound intensity does not exceed 3 W/cm^2; the acoustic Reynolds number is less than 2, and the nonlinear distortions of the wave shape are slight. The near-field nonuniformity, which is typical of acoustic transducers, prevents manifestation of the self-focusing of sound at distances less than the length of the projector zone, $\pi d^2 f/4c \sim 30$ cm. The amplitude and phase measurements are accordingly carried out in the far zone, where the field oscillations along and across the beam axis are smoothed, but the wavefront is curved as a result of the spherical divergence.

The sound is received by a wide-band hydrophone at a fixed position. In the weak-signal case, a phase shift $\varphi = 0$ is established between the voltages on the transducer and the receiver. The phase shift $\varphi = 0$ (within 1.5°) is achieved by adjusting the frequency (within 30 Hz). The voltage on the transducer is then increased abruptly. Working from the Lissajous figures, we determine φ and calculate the displacement of the wavefront, $\Delta = \lambda\,|\varphi|/360°$. Since the wavefront always moves toward the transducer (with respect to its original position, at the low sound power), the thermal self-focusing of sound overpowers the competing self-effect from convection.

4. We observed that the wavefront moves toward the transducer when the intense

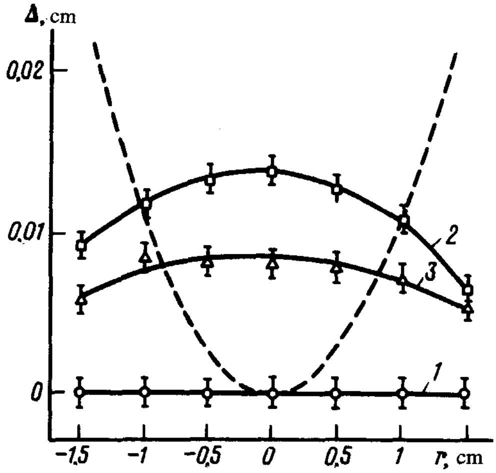

FIG. 1. The displacement $\Delta(r)$ of the wavefront due to the self-effect in transformer oil. The distance from the transducer is $x = 45$ cm, and the power is 8 W. Curves 1–3—Measurements at times 0, 20, and 40 s after the ultrasound is turned on; dashed curve—the unperturbed wavefront, $\Delta = r^2/2x$.

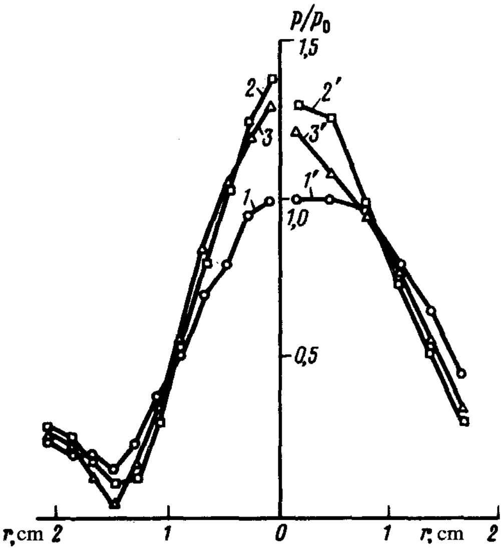

FIG. 2. Change in the profile of the pressure p in the beam during the self-focusing. Here p_0 is the pressure at the axis at $t = 0$. Left: Glycerin, $x = 30$ cm. Right: Oil, $x = 45$ cm. The power is 12 W. Curves 1 (1'), 2 (2'), 3 (3')—Measured at $t_1 = 0$, $t_2 = 50$ s (15 s), and $t_3 = 100$ s (40 s). Here t_2 is the time at which the self-focusing is at its greatest.

ultrasound is turned on. This displacement reaches a maximum and then falls back to a constant value. At the periphery of the beam the displacement is less than at the axis. As a result, the front acquires a steady-state shape with a partial flattening (Fig. 1).

The change in the curvature of the wavefront is accompanied by an increase in the wave amplitude at the beam axis. At a power $W > 5$ W, this increase reaches 40%. The amplitude rises to a maximum and then falls off to a level exceeding the original level. At the periphery of the beam, we observe the opposite pattern—the amplitude decreases—and the beam contracts (Fig. 2). The time scale for the onset of the self-focusing (the time over which the maximum increase in the amplitude at the axis is reached) is found to have the behavior $\tau \sim W^{-1}$.

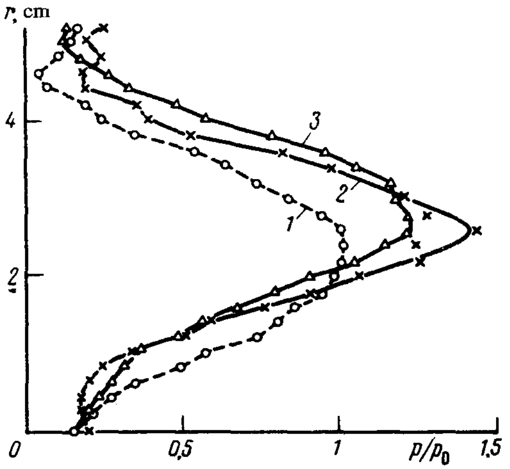

FIG. 3. "Upward floating" of the sound beam caused by convection. These measurements were taken in glycerin at a distance of 40 cm, at the same times and at the same sound power levels as in Fig. 2.

5. The experimental results can be interpreted in a qualitative way in the theory of Refs. 3 and 4 for the self-focusing of sound. The critical parameter is the wave energy

$$E_{\mathrm{cr}} = \frac{(0{,}61\,\lambda)^2\,\pi\rho c_p}{16\,\alpha\,|c^{-1}\,dc/dT|},$$

where ρ and c_p are the density and specific heat of the liquid, and c and α are the velocity and absorption coefficient of the sound. The energy corresponding to the maximum self-focusing in the present experiments is 200 J (for the oil) or 800 J (for the glycerin), substantially higher than the theoretical prediction (150 and 130 J). This difference, which is more important in the case of glycerin, stems from the pronounced energy loss in the near field. In addition, the time scale over which the convection develops is not greatly different from the time required to reach the threshold for the self-focusing of sound ($\sim$10 s). Because of these factors, we are not able to achieve a high field intensification at the axis, and in the vertical plane we see the beam "float upward" (Fig. 3). (Here we are seeing basically a thermal convection, as in nonlinear optics.[7]) The nonmonotonic time evolution of the wave amplitude and phase is explained on the basis that the acoustic and thermal convection currents arise more slowly than the self-focusing of the sound.

These results are evidence that we have in fact observed a self-focusing of sound.

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