Highly nonequilibrium localized states in systems slightly away from thermodynamic equilibrium

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In an intrinsic semiconductor in a very weak electric field, a brief local perturbation may excite a stable localized state with a reduced carrier density and a high carrier temperature. The amplitude of this state will increase with decreasing strength of the electric field in which the effect occurs.

For simplicity, we consider a relatively narrow-gap semiconductor with equal electron and hole properties ($m_e^* = m_h^*$, etc.) and with an intrinsic-carrier density ($n = p = n_i$) at lattice temperatures $T_0 > \theta_D$ (θ_D is the Debye temperature) such that the conditions $\tau_p \ll \tau_{ee} \ll \tau_\epsilon$ hold (τ_p, τ_ϵ, and τ_{ee} are the relaxation times for momentum, energy, and electron-electron collisions). These conditions hold, for example, in PbTe at $T_0 \sim 400$ K, at which we have $n_i \sim 10^{17}$ cm^{-3}. A quasi-neutral electron-hole plasma of this sort in a uniform electric field **E** is described by[1]

$$\tau_r \, \partial n / \partial t = L^2 \partial^2 / \partial x^2 \, [nD(T)/D^0] - (n - n_i) , \tag{1}$$

$$\frac{3}{2} \tau_\epsilon^0 \frac{\partial (nT)}{\partial t} = l^2 \frac{\partial^2}{\partial x^2} \left[\frac{n D(T) T}{D_0} \right] + \frac{j^2 \tau_\epsilon^0}{4\sigma} - n \frac{T - T_0}{T^s} T_0^s , \tag{2}$$

where n, T, $D(T) = \mu T / e$, σ, and τ_r are the density, temperature, diffusion coefficient, conductivity, and lifetime of the carriers; $l = (D^0 \tau_\epsilon^0 (5/2 + \alpha))^{1/2}$ and $L = (D^0 \tau_r)^{1/2}$ are the energy relaxation length and the carrier diffusion length; j is the current density; and the superscript zero specifies the equilibrium value of the corresponding quantity ($T = T_0$, $n = n_i$). In writing Eqs. (1) and (2) we have used $\tau_p \propto T^\alpha$, $\tau_\epsilon \propto T^S$; by virtue of the symmetry of the electron-hole plasma, the ambipolar field in it is zero, and the electron component of the current is

$$j_e = n (1 + \alpha) \mu \, \partial T / \partial x + eD \partial n / \partial x + j/2 = e \, \partial / \partial x \, (nD(T)) + j/2. \tag{3}$$

The effect in which we are interested here occurs if

$$L \gg l \quad \text{and} \quad \alpha + s > -1. \tag{4}$$

With decreasing $\epsilon = l/L$, a solitary eigenstate of the electron-hole plasma —an autosoliton—can be excited at a progressively lower current in the semiconductor, and the amplitude of this soliton, i.e., the electron temperature at the center of the soliton, T_m, will be progressively higher. An important property of this electron-hole plasma, in contrast with that studied in Refs. 1–3, is that condition (4) holds in it up to high values of T, because at $T > \theta_D$ the time τ_ϵ increases with increasing[4] T. In nonpolar semiconductors such as Ge and Si, and at $T > \theta_D$, we have $s = 1/2$ and $\alpha = -1/2$, i.e., $\alpha + s = 0$. In polar semiconductors, the conditions[4] $0 \leqslant \alpha + s \leqslant 1$ usually hold.

We write Eqs. (1) and (2) for the steady state as

$$d^2\eta/dx^2 - \epsilon^2 Q(\eta,\theta) = 0,\ Q = \eta\theta^{-1-\alpha} - 1;\qquad \eta = (n/n_i)\theta^{1+\alpha}, \tag{5}$$

$$d^2(\eta\theta)/dx^2 - q(\eta,\theta,W) = 0,\ \ q = \eta\theta^{-1-\alpha-s}(\theta-1) - W\theta\eta^{-1}, \tag{6}$$

where distances are expressed in units of l; $\theta = T/T_0$; and $W = j^2\tau_\epsilon^0/4\sigma^0 n_i T_0$ is the dimensionless power. In the zeroth approximation[3,4] in ϵ, Eqs. (5) and (6) have a solution $T(x)$, $n(x)$ in the form of an autosoliton (the dashed lines in Fig. 1), which correspond to the separatrix of the equation

$$d^2\theta/dx^2 - q(\eta_h,\theta,W) = 0, \eta_h = \theta_h^{1+\alpha};\ \ n(x) = n_i[T_h/T(x)]^{1+\alpha}, \tag{7}$$

which is closed at $x \to \pm\infty$ at the saddle point $\theta = \theta_h = T_h/T_0$, which corresponds to a homogeneous state of an electron-hole plasma with $n = n_i$ and $T = T_h$. Such a separatrix satisfies the condition

$$\int_{\theta_h}^{\theta_m} q\,d\theta = \int_{\theta_h}^{\theta_m} \{\eta_h\theta^{-1-\alpha-s}(\theta-1) - W\theta\eta_h^{-1}\}d\theta = 0, \tag{8}$$

from which we easily find a transcendental equation for $\theta_m = T_m/T_0$. It follows from this equation that under the condition $\alpha + s = 0$ we have $\theta_m \simeq 2/W$, while at $\alpha + s = 1$ we have $\theta_m(\ln\theta_m)^{1/2} \simeq (2/W)^{1/2}$. In other words θ_m increases with decreasing W. Using the results of Refs. 2 and 3, we can take into account the corrections of higher orders in ϵ, and we can show that the autosoliton acquires a more complicated shape (the solid lines in Fig. 1); by virtue of the condition $\theta_m \gg \theta_h$, the soliton is stable. These conclusions are supported by a numerical solution of the complete system of

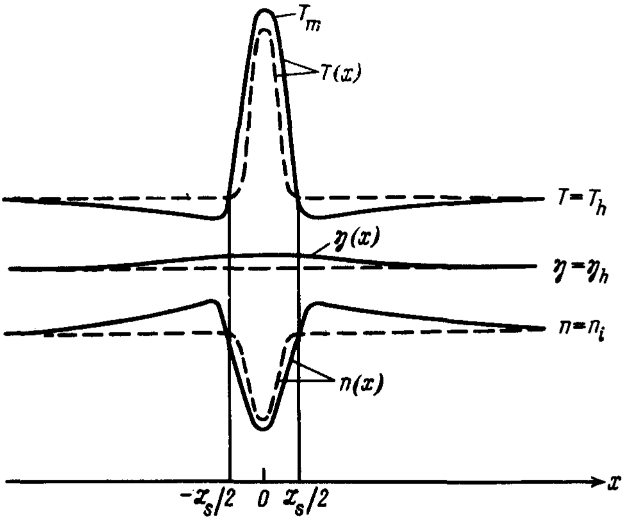

FIG. 1. Structure of an autosoliton in an electron-hole plasma.

equations (1) and (2), which also yield information on the process by which the autosoliton is formed. The results show that the autosoliton forms only if the amplitude of the brief initial local perturbation exceeds a certain critical value; otherwise, the perturbations decay to a homogeneous state.

The reason for the existence of an autosoliton (Fig. 1) is that there is an intense ejection of hot carriers from the hot region because of thermodiffusion; as a result, the density of these carriers and thus the conductivity σ at the center of the autosoliton decrease (Fig. 1). On the one hand, this effect maintains a high value of T at the center of the autosoliton, since it increases the power $W = j^2/\sigma$ going from the electric field to the carriers. On the other hand, the density dip in the autosoliton does not spread, because the diffusion flux is balanced by a thermal flux (Fig. 1). This conclusion—i.e., the conclusion that the sum of the first and second terms in (3) is approximately zero in an autosoliton of dimension $\mathscr{L}_s \ll L$—follows from Eq. (1), according to which $j_e(x)$ varies over a distance on the order of L, and we have $\eta \equiv D(T)n/D^0 n_i \simeq$const. With $\mathscr{L}_S \gg l$, we may assume that the energy balance equation (2) holds locally. Since $\eta \propto D(T)n \simeq$const and $T_m \gg T_0$, we then find

$$T_m \propto W^{-(1+\alpha+s)^{-1}}, \quad (T_h - T_0)/T_0 \simeq (T_0/T_m)^{1+\alpha+s} \ll 1. \tag{9}$$

In other words, in accordance with the conclusions reached from (8), the amplitude of the autosoliton increases with decreasing W under conditions (4). In turn, the minimum value $W = W_b$, at which an autosoliton still exists, decreases with decreasing $\epsilon = l/L$. The reason is that the dimension of the autosoliton, $\mathscr{L}_s$, increases with increasing T_m because of the finite thermal conductivity of the electron gas. At low values of W, the quantity $\mathscr{L}_s$ thus reaches values on the order of L, limiting the value of W_b. These conclusions are supported by the numerical calculations, which show that small values of $\epsilon = l/L$ correspond to small values of W_b, i.e., to low heating levels $T_h = T_b$ of the homogeneous electron-hole plasma in which an autosoliton can be excited.

The properties of several systems, including low-temperature gaseous plasmas, are described by equations like (1) and (2). We thus do not rule out the possibility that an autosoliton—a stable, highly nonequilibrium localized eigenstate— can be excited in many other systems which are nearly at equilibrium, e.g., the weakly ionized atmosphere. In other words, we do not rule out the possibility that ball lightning is an autosoliton excited by a brief and intense local perturbation in the weakly ionized atmosphere.

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[1]B. S. Kerner and V. V. Osipov, Zh. Eksp. Teor. Fiz. **71**, 1542 (1976) [Sov. Phys. JETP **44**, 807 (1976)].
[2]B. S. Kerner and V. V. Osipov, Zh. Eksp. Teor. Fiz. **74**, 1675 (1978) [Sov. Phys. JETP **47**, 874 (1978)].
[3]B. S. Kerner and V. V. Osipov, Fiz. Tekh. Poluprovodn. **13**, 721 (1979) [Sov. Phys. Semicond. **13**, 424 (1979)].
[4]E. M. Conwell, High Field Transport in Semiconductors, Suppl. 9 to Solid State Phys., Academic Press, New York (1977) (Russ. transl. Mir, Moscow, 1970).

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