Quantization of the Faraday effect in systems with a quantum Hall effect

V. A. Volkov and S. A. Mikhaĭlov
Institute of Radio Engineering and Electronics, Academy of Sciences of the USSR

(Submitted 25 March 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 9, 389–390 (10 May 1985)

The angle of rotation of the polarization plane of low-frequency electromagnetic radiation, φ, transmitted through a two-dimensional ($2D$) electronic layer, which is in a regime corresponding to an integer or fractional quantum Hall effect, is found to be quantized. In certain cases $\Delta\varphi = e^2/\hbar c = 1/137$ deg.

The quantum Hall effect, which has been observed[1,2] at low temperatures in $2D$ electron systems in quantizing magnetic fields H_0, involves a quantization of the static Hall conductivity σ_{xy} and the vanishing of the diagonal conductivity σ_{xx}: over finite intervals of H_0 or electron densities.

$$\sigma_{xy}(0) = \frac{e^2}{2\pi\hbar}\nu, \qquad \sigma_{xx}(0) \approx 0. \tag{1}$$

Here ν is an integer or a fractional number (integer or fractional quantum Hall effect, respectively).

Let us consider the Faraday effect in systems which have a conductivity of the type in (1) in the low-frequency limit. This feature sets the formulation of this problem apart from that in Ref. 3, where a Drude-type mode was used. We will first examine a very simple case, in which a $2D$ electronic layer in a vacuum occupies the Oxy plane. A linearly polarized $\mathbf{E} = (0, E_y, 0)$ electromagnetic wave of frequency ω is incident along the normal (the z axis). The external magnetic field is $\mathbf{H} = (0,0,H_0)$. The alternating field of a wave, E_y, induces an alternating current $j_x = \sigma_{xy}E_y$ in the $2D$ layer, which generates on either side of the layer an electromagnetic wave with a magnetic vector $H_y = 2\pi j_x/c$ and an electric vector $E_x = E_y$, where c is the speed of light. This wave in turn again induces a current, etc. The angle of rotation of the polarization plane of the transmitted wave, φ, and its ellipticity δ are determined by the real part (σ'_{xy}) and the imaginary part (σ''_{xy}) of σ_{xy}:

$$\tan\varphi = \mathrm{Re}(E_x/E_y) = 2\pi\sigma'_{xy}(\omega)/c, \tag{2}$$

$$\delta = \mathrm{Im}(E_x/E_y) = 2\pi\sigma''_{xy}(\omega)/c. \tag{3}$$

The corrections to (2) and (3) are small relative to the parameter $2\pi\sigma_{xx}/c \ll 1$. Since $\sigma'_{\alpha\beta}$ is an even function and $\sigma''_{\alpha\beta}$ is an odd function of ω, we have $\sigma_{xy}(\omega) \approx \sigma_{xy}(0)$ in the low-frequency limit. From (1) and (2) we find the law governing the φ quantization of the transmitted wave in the limit $\omega \to 0$:

$$\tan\varphi = \frac{e^2}{\hbar c}\nu, \tag{4}$$

where $e^2/\hbar c \approx 1/137$ is the fine-structure constant, and ν is an integer or a fractional number. Accordingly, the angle φ increases as a result of multiple transmissions of the wave through a $2D$ layer (in a superlattice, for example). The transmission coefficient T and the reflection coefficient R are also quantized with respect to the intensity:

$$T = 1 - R = \left[1 + \left(\frac{e^2 \nu}{\hbar c}\right)^2\right]^{-1} .$$

In general, φ is determined by the refractive index of the surrounding medium. Equation (4) is nonetheless approximately correct if the sample's thickness d is small in comparison with the emission wavelength λ. The sample is generally a multilayer structure, and the field of the wave can be determined from the solution of the wave equation

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} + \frac{n^2 \omega^2}{c^2} \mathbf{E} + \frac{4\pi i \omega}{c^2} \mathbf{j}\, \delta(z) = 0 \tag{5}$$

with the usual boundary conditions; $j_\alpha = \sigma_{\alpha\beta} E_\beta$. We will give the solution of Eq. (5) for the most important cases.

In an infinite medium with a refractive index n we have

$$\tan \varphi = \mathrm{Re}\left(\frac{2\pi \sigma_{xy}/c}{n + 2\pi \sigma_{xx}/c}\right) \tag{6}$$

for a $2D$ layer. In the limit $\omega \to 0$, the solutions of (6) and (1) differ from the solution of (4) by a factor $1/n$.

For the structure vacuum (refractive index 1)—$2D$ layer—substrate (thickness d, refractive index n)—vacuum, we find

$$\tan \varphi = \frac{2e^2 \nu}{\hbar c}\left[\frac{2n^2 - (n^2 - 1)\sin^2(\omega n d/c)}{4n^2 + (n^2 - 1)^2 \sin^2(\omega n d/c)}\right], \tag{7}$$

$$\delta = \frac{2e^2 \nu}{\hbar c}\left[\frac{n(n^2 - 1)\sin(\omega n d/c)\cos(\omega n d/c)}{4n^2 + (n^2 - 1)^2 \sin^2(\omega n d/c)}\right] \tag{8}$$

in the limit $\omega \to 0$. The quantized values in (7) and (8) oscillate because of the change in the wavelength $\lambda = 2\pi c/n\omega$. At resonance, when $2d/\lambda$ is an integer, we find $\delta = 0$, while (7) transforms to (4) and does not depend on the properties of the substrate. Analogous effects evidently occur in the case of oblique incidence and in the case of a reflected wave. We will omit the general expressions, which are rather lengthy.

The principal approximation in this case involves a replacement of $\sigma_{\alpha\beta}(\omega)$ by its static value (1). The value of ω in this case must be small in comparison with the scale frequency ω_0. The scale frequency ω_0, which is low in comparison with the cyclotron frequency of an integer quantum Hall effect (or in comparison with the size of the gap for a fractional quantum Hall effect), depends on the position of the Fermi level with respect to the Landau levels. The model-based calculations[4,5] yield for ω_0 a value on the order of the scale amplitude of the impurity potential. For typical parameter values the permissible values of ω are in the centimeter or millimeter range. The microwave Faraday effect, as we know,[6] can be measured quite accurately ($\Delta\varphi \simeq 10^{-4}$ deg).

The effect under consideration, which by analogy with the quantum Hall effect can be called the quantum Faraday effect, cannot compete with the quantum Hall effect in determining the universal constants. The quantum Faraday effect can nonetheless yield information which would be difficult or impossible to extract from the quantum Hall effect. 1) The quantum Faraday effect is governed by the conductivity δ_{xy}, whereas the quantum Hall effect is governed by the resistance ρ_{xy}. This may be an important factor, for example, in the search for the secondary quantization of σ_{xy} in weak H_0, which was predicted by Khmel'nitskiĭ.[7] 2) A divergence from the quantum Faraday effect restores the dispersion $\sigma'_{xy}(\omega)$ and allows us to study the contribution of various excitations to this dispersion, in particular, the "fractional" excitons.[8] 3) The quantum Faraday effect can be measured by a contactless method, which may be an important asset in systems with large-scale fluctuations.

We wish to thank V. B. Sandomirskiĭ and G. R. Aĭzin for useful discussions, and V. N. Solyakov for a discussion of the capabilities of the microwave measurements.

[1]K. Von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[2]D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[3]R. F. O'Connell and G. Wallace, Phys. Rev. B **26**, 2231 (1982); B **28**, 4643 (1983).
[4]S. M. Apenko and Yu. E. Lozovik, Preprint FIAN, No. 196, 1, 1984.
[5]Yu. E. Lozovik, V. M. Farztdinov, and Zh. S. Gevorkyan, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 153 (1984) [JETP Lett. **39**, 179 (1984)].
[6]S. Chai and P. Vogelhut, Rev. Scient. Instr. **37**, 1620 (1966).
[7]D. E. Khmel'nitskiĭ, Phys. Lett. **A106**, 182 (1984).
[8]R. B. Laughlin, Physica **126B**, 254 (1984).

Translated by S. J. Amoretty