Surface conductivity of germanium and gauge theory of localization

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The experimental results on the conductivity of cleaved germanium surfaces and germanium bicrystals can be described by a single-parameter gauge theory of localization.

Davies *et al.*[1] have recently proposed a method for testing the hypothesis of gauge invariance through an experimental determination of the quantity $\beta_e = -d \ln \sigma / d \ln T$ as a function of the conductivity (σ) of a system near the metal-insulator transition at low temperatures.

The quantity β_e is directly related to the function $\beta(g) = d \ln g / d \ln L$ in the equation of the single-parameter gauge theory of localization,[2] where $g = (\hbar/e^2)\sigma L^{d-2}$ is the dimensionless conductivity, $\hbar$ is Planck's constant, e is the electron charge, L is the linear dimension of the system, and $\beta(g)$ is a universal function which depends only on the dimensionality of the system, d. Under conditions such that the phase shift of the wave function is determined by inelastic scattering, the role of L is played by the typical diffusion distance over the time interval between inelastic collisions, L_{in}. If we can assume $L_{in} \sim T^{-\gamma}$, then we have $\beta_e = \gamma(2 - d + \beta)$, where γ is a numerical factor on the order of unity. In the 2D case, the relation between these functions takes a particularly simple form:

$$\beta_e = \gamma\beta,$$

while the dimensionless conductivity is $g \simeq 4k\Omega\sigma_{\square}$. If the hypothesis of single-parameter scaling is valid, we can thus work from measurements of the conductivity $\sigma(T)$ of 2D systems to find the universal function β_e, which is proportional to the scaling value β.

Attempts were made in Refs. 1 and 3 to find the dependence $\beta_e(g)$, but the results were contradictory: The function β_e was not a universal function in the case of inversion layers at a Si–SiO_2 interface, while for the 2D systems studied in Ref. 3 the functional dependences $\beta_e = f(g)$ turned out to agree well with each other and with the results of numerical calculations.[4]

In the present study we have determined the functions $\beta_e = -d \ln g / d \ln T$ from measurements of the electrical conductivity in two germanium systems with a 2D hole conductivity: cleaved crystal surfaces and intergrowth surfaces of bicrystals. Figure 1 shows the results on $\beta_e = f(g)$. The first curve, $\beta_e(1)$, was found from an analysis of experimental results on $\sigma(T)$, shown by curves 1–18 in Fig. 2 in Ref. 5, for cleaved crystal faces obtained by cleavage in liquid helium, followed by heat treatment in

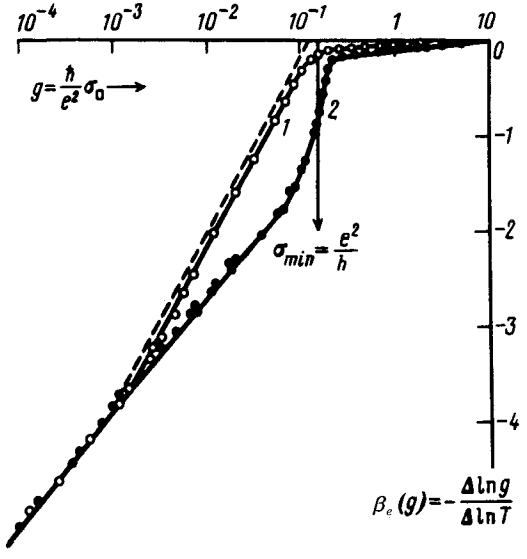

FIG. 1. The functions $\beta_e = -(\Delta \ln g/\Delta \ln T)$ found from measurements of the electrical conductivity. 1—At cleaved Ge faces at $T = 1.1$–1.5 K; 2—for Ge bicrystals in the region $0.3 \leqslant T < 5$ K. The dashed line shows the function $\beta(g)$ from Ref. 4.

saturated helium vapor. The second curve, $\beta_e(2)$, was found from the curves of $\sigma = f(T)$, shown by curves 1–11 in Fig. 2 in Ref. 6 and referring to germanium bicrystals with tilt angles $7° \leqslant \theta \leqslant 25°$.

We see that at both small and large values of g the functions $\beta_e(g)$ do not depend on the particular features of the systems; they are of a universal form which agrees with the results of the gauge theory of localization.[2] At small values of g, in the hopping-conductivity region, the slope of the rectilinear parts of the $\beta_e(\ln g)$ curves in Fig. 1 is approximately 1/2, in agreement with the theoretical asymptote if we take the inelastic length L_{in} to be the hopping length and if we assume $\gamma = 1/2$. At large values of g, the conductivity depends logarithmically on the temperature, in accordance with calculations of the localization and correlation corrections to the conductivity of 2D systems.[2,7] However, in the intermediate region, $g \sim 0.1$, corresponding to a transition from a logarithmic temperature dependence of the conductivity to an exponential

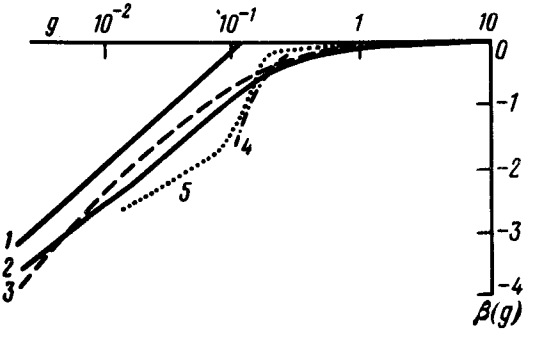

FIG. 2. The functions $\beta = f(g)$. 1—From Ref. 4; 2—Ref. 8; 3—Ref. 9; 4—the function $\overline{\beta}(\overline{g})$ with $g^* = 4$; 5—the function $\beta_e(2)$. Here $g^* = S((\hbar/e^2)\sigma_B - \ln(2b/l))$, where σ_B is the conductivity found from the kinetic equation, and l is the mean free path with respect to elastic collisions.

dependence, curves 1 and 2 in Fig. 1 do not agree with each other or with the calculated results of Refs. 8 and 9.

Figure 2 shows curves of $\beta = f(g)$ derived in the self-consistent theory of localization[8] and through a smooth single-parameter interpolation,[9] along with the results of numerical calculations of $\beta(g)$ for the Anderson model.[4] We see that the calculated results of Ref. 4 agree qualitatively with the $\beta_e(1)$ curve, found from the measurements of $\sigma(T)$ on cleaved surfaces of Ge crystals, while the theory of Refs. 8 and 9 predicts a slower increase in $\beta(g)$ in the transition region of g. (The latter result may be a consequence of effects of the type discussed in Ref. 10, which lie outside the scope of the present study.)

As for the curve $\beta_e(2)$, we see from Fig. 1 that it drops extremely sharply at $g \sim 0.1$ and falls off quite smoothly with a further decrease in g. The observed features of the function $\beta_e(2)$ are evidently due to the particular structure of the intergrowth surfaces of bicrystals. As was mentioned in Refs. 6 and 11, in the region of the nonmetallic conductivity, the electrical conductivity of bicrystals is very anisotropic. Both the coefficient of the exponential function, σ_0, and the parameter T_0 in the functional dependence $\sigma(T) = \sigma_0 \exp[-(T_0/T)^{1/2}]$ are anisotropic. The theories of the hopping conductivity for anisotropic localized states[12] and for an anisotropic distribution of local centers predict an anisotropy of σ_0 alone. The experimentally observed anisotropy of the argument of the exponential function in the expression for the conductivity can be explained by assuming that the bicrystal intergrowth surfaces studied in Refs. 6 and 11 contain macroscopic inhomogeneities stretched out along rows of edge atoms. The longitudinal conductivity may be determined by channels with relatively high values of σ, while the transverse conductivity is determined by the regions between channels, since there is no percolation along channels with a relatively high conductivity in the transverse direction. Because of dimensionality effects, such longitudinal inhomogeneities can significantly affect the function β_e in the region $g \sim 0.1$ if the length L_{in} is comparable to the scale width of the percolation channels. We have calculated the function $\overline{\beta} = d \ln \bar{g}/d \ln L_{in}$ for a model 2D system (of conducting bands) by the method of Refs. 13 and 14 for calculating localization corrections. The results are shown by the dot-dashed curve in Fig. 2, where $\bar{g}$ is the effective dimensionless conductivity, and S is a geometric factor, which is a measure of the fraction of the area occupied by conducting bands. We see that in the transition region along the g scale the curve of $\overline{\beta}(\bar{g})$ for the model system of bands agrees qualitatively with the curve $\beta_e(2)$, found from measurements of $\sigma(T)$ of bicrystals. This correspondence between the functions $\overline{\beta}$ and $\beta_e(2)$ suggests that the transition from a metallic conductivity to an activated conductivity in Ge bicrystals results from a decrease in the effective dimensionality of the conducting system and a transition from a 2D conductivity to a quasi-1D conductivity at values $g \sim 0.1$.

With a further decrease in g, in the region of the hopping conductivity, the effective dimensionality of the surface of the bicrystals may again increase because of the "two-dimensionalization" of the paths traced out by the successive hops of charge carriers. We know that in 1D systems the topological restrictions on the shape of the paths in the hopping-conductivity region[15] lead to a decay of $\sigma = f(T)$ which is steeper than in the case of 2D systems. As a result, the optimum paths may turn out to be

paths which do not run along 1D channels but are instead 2D paths, despite the fact that they pass through regions with larger values of the parameter T_0. Under these conditions, there could, in principle, be deviations of the function $\beta_e(2)$ from a universal form, but such deviations have not been observed for the bicrystals studied.

In summary, these experimental results on the conductivity of cleaved surfaces and bicrystals of germanium can be described by a single-parameter gauge theory of localization. The observed features of the function $\beta_e(2)$ in the region $g \sim 0.1$ are consistent with the predictions of this theory, and they can be attributed to a change in the effective dimensionality of the system (2) in this region of g.

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