Analog of the Aharonov-Bohm effect in superfluid He^3-A

M. V. Khazan
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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An analogy is found between the dynamics of the collective clapping mode near a vortex with a half-integer number of circulation quanta ($n = 1/2$) in He^3-A and the dynamics of an electron outside a cylindrical tube with the magnetic flux. Because of the Aharonov-Bohm effect, the scattering of the clapping mode by vortices with $n = 1/2$ should be substantially greater than the scattering by vortices with $n = 1$ or 2. This difference might be exploited to identify vortices.

The vortex textures that arise during the rotation of He^3 have recently become the subject of active research. Experiments of the A phase of He^3 have revealed[1] nonsingular vortices with two circulation quanta: $n = 2$. In addition, there can be single vortices with $n = 1$ and so-called half-vortices with $n = 1/2$. The latter are energetically preferred to vortices with $n = 1$ or 2 in a parallel-plane geometry with a magnetic field perpendicular to the planes.[2] The basic method for studying vortex textures is to study their effects on the dynamics of collective modes propagating in the volume of the liquid. These modes are oscillations of the order parameter—the matrix $A_{\alpha i} = A^0_{\alpha i} + \delta A_{\alpha i}$—with respect to the equilibrium value $A^0_{\alpha i} = d_\alpha(\mathbf{r})(\Delta'_i + i\Delta''_i)(\mathbf{r})$, which depends on the coordinates (texture). Here d_α is a unit vector in the spin space, orthogonal to the spin of the pair, and Δ'_i, Δ''_i and $\mathbf{l}$—the moment of the pair—form a triad in orbital space. Previous studies of vortices have used collective modes such as spin waves, which have been excited in NMR experiments. Ultrasonic spectroscopy is used to study textures in He^3-A and will soon be used to study vortex textures.

Experiments[3] and the theory of Ref. 4 show that a promising method for studying textures is to use other collective modes of the order parameter, such as the "real squashing" mode in He^3-B. In the present letter we show that for an identification of the vortices in He^3-A it is convenient to use one of the so-called clapping modes. In the overall classification[5] of collective modes in He^3-A, this mode corresponds to the quantum numbers, $S_d = 0$, $Q = 2$. This mode has a spectrum with a gap, $\omega^2 = \omega_0^2 + c^2q^2$, where $\omega_0 = 1.22\Delta(T)$, [$\Delta(T)$ is the gap in the spectrum of Fermi excitations], $c \sim v_F$, and a natural width $\Gamma_{cl} = \beta\omega_0$ (near T_c, we have $\beta \approx 0.1$).

We show below that an analog of the Aharonov-Bohm effect should cause the scattering of the clapping mode by a vortex with $n = 1/2$ to be stronger than the scattering by vortices of other types. The clapping mode with the quantum numbers $S_d = 0$ and $Q = 2$ is described by the complex scalar ψ:

$$\delta A_{\alpha i} = \psi d_\alpha (\Delta'_i - i\Delta''_i). \tag{1}$$

In a short-wavelength mode, ψ varies rapidly against the background of a rapidly varying texture of the vectors $\mathbf{d}$ and $\vec{\Delta}' + i\vec{\Delta}''$.

Let us examine the scattering of the clapping mode by the vortex texture, which arises in the parallel-plane geometry in a magnetic field perpendicular to the planes (these conditions favor the formation of vortices with $n = 1/2$). In this geometry, the field of the order parameter A^0 near a vortex with n quanta can be written

$$d_\alpha = \begin{cases} \text{const}, & n = 1, 2 \\ \hat{x}_\alpha \cos\varphi/2 - \hat{y}_\alpha \sin\varphi/2, & n = 1/2 \end{cases} \tag{2}$$

$$\Delta_i' + i\Delta_i'' = (\hat{x}_i + i\hat{y}_i)\exp(in\varphi),$$

where φ is the azimuthal angle in the plane, and $\hat{x}_\alpha, \hat{y}_\alpha$ and $\hat{x}_i, \hat{y}_i$ are the unit vectors in the spin and orbital spaces. The dynamics of the clapping mode near vortex (2) can be described in terms of Lagrangian which is quadratic in $\delta A_{\alpha i}$ in (1) and which is written as follows, by virtue of symmetry considerations:

$$\mathcal{L} = (\omega_0^2 - \omega^2)|\delta A_{\alpha i}|^2 + b_1|\nabla_k \delta A_{\alpha i}|^2 + b_2|\nabla_i \delta A_{\alpha i}|^2. \tag{3}$$

Varying $\mathscr{L}$ with respect the ψ, we find a two-dimensional wave equation for $\psi(x,y)$, which is axisymmetric in this particular geometry:

$$c^2(\vec{\nabla} - i\mathbf{A})^2\psi + U\psi = (\omega_0^2 - \omega^2)\psi. \tag{4}$$

Here the vector $\mathbf{A} = \Delta_i' \vec{\nabla} \Delta_i''$ serves as an Abelian gauge field for the collective mode; upon rotation through an angle α around the direction $\mathbf{l} = [\vec{\Delta}'\vec{\Delta}'']$, this vector transforms in accordance with

$$\mathbf{A} \to \tilde{\mathbf{A}} = \Delta_i' \vec{\nabla} \Delta_i'' - \vec{\nabla}\alpha.$$

The function ψ should transform in accordance with

$$\psi \to \psi e^{-i\alpha},$$

according to (1), and $U = -c^2(\nabla_k d_\alpha)^2$ plays the role of an ordinary potential. Equation (4) is analogous to the Schrödinger equation for an electron outside a cylindrical tube with a magnetic flux (the role of this tube is played here by the core of the vortex, with a dimension that depends on the geometry of the system; this dimension would be either the dipole length $\xi_D \sim 10^{-3}$ cm in an open geometry or the distance between plates, a, in the case $a < \xi_D$). Outside this region, we have $\mathbf{A} = (n/(x^2 + y^2)^{1/2})\hat{\varphi}$ and $\text{curl}\mathbf{A} = 0$. The scattering problem for Eq. (4) was solved originally in Ref. 6. The scattering through small angles θ is dominated by the Aharonov-Bohm effect due to the existence of a vector potential in a multiply connected region outside the vortex core:

$$d\sigma = \frac{\sin^2 \pi n}{2\pi q} \frac{d\theta}{\sin^2 \theta/2},$$

where q is the momentum of the excitations. We see that the effect occurs only for vortices with a half-integer number of circular quanta, i.e., for $n = 1/2$. Because of the Aharonov-Bohm effect, the cross section for scattering of the clapping mode by vortices with $n = 1/2$ is substantially larger than the cross section for scattering by vortices of other types. This difference might be exploited to identify vortices with $n = 1/2$.

No direct method for exciting the clapping mode, analogous to the use of periodic heat pulses to excite second sound in He II, is presently known. In the present experiments we accordingly use ultrasound to excite this mode (the excitation condition is $c_0 q = \omega_{cl}$, where c_0 is the zero-sound velocity, $c_0 \gg v_F$). This excitation method gives rise to a resonant peak on the sound absorption curve. Because of the scattering of excitations, the vortices cause an additional broadening (γ) of the absorption line:

$$\frac{\gamma}{\Gamma_{cl}} = \frac{v_{gr} N_v \sigma}{\beta \omega_{cl}} \sim \frac{1}{\beta} (c/c_0)^2 \frac{(N_v)^{1/2}}{q}, \qquad \theta_{min} \sim \frac{1}{q r_v},$$

where $v_{gr} = \partial\omega/\partial q$ is the group velocity of the mode, N_v is the density of vortices (r_v is the distance between vortices), and σ is the scattering cross section. Estimates show that at low pressures, and at temperatures not too far from T_c, this broadening is $\sim 10\%$ of the natural line width, even in the presence of an equilibrium density of vortices, $N_v \sim 10^4$ cm^{-2}, in the rotating vessel. If, on the other hand, the half-vortices are excited through a dissipation of a superfluid motion, their density may be two or three orders of magnitude greater, and the effect will be seen even at high pressures, where c/c_0 is low. This method can be used to observe the phase transition from vortices with $n = 2$ to vortices with $n = 1/2$ as the temperature is changed in experiments with a countercurrent.

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