Random inflation and global geometry of the universe

V. A. Berezin, V. A. Kuz'min, and I. I. Tkachev
Institute of Nuclear Research, Academy of Sciences of the USSR

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Either an inflationary fluctuation in the early universe is a mole's hole (this is possible in either a closed or open universe) or the fluctuation occupies more than half of a closed universe. Other possibilities are extremely exotic.

Bubbles that arise in cosmological phase transitions have recently attracted considerable interest.[1–9] These bubbles may be bubbles of a new phase in the interior of an old phase[1–3,5–9] or remnants of an old phase surrounded by a new phase.[4,5,9] Clearly, effects of the general theory of relativity must be considered in the study of such entities.[3–6,9] Theories of an inflationary universe are essentially also theories of the evolution of bubbles,[10–12] even if the entire visible part of the universe lies inside a single bubble.[11,12] Bubbles can be studied by the method of thin shells[13] which was developed in Ref. 5 for the case of bubbles that arise upon cosmological phase transitions.

The matter in the inflationary region is described by a vacuum equation of state. In the scenario of a random universe,[12] a region occupied by a fluctuation of a scalar field inflates (the pressure in this region is less than zero: $p<0$). This region is surrounded by a phase with a higher pressure $p_{out} > p_{in}$; in particular, $p_{out} > 0$. Such a situation might arise in the scenario of Ref. 11 also, if percolation through the new phase in the corresponding phase transition sets in before the conditions for inflation are attained.[14] Clearly, if gravitational effects are small, such a bubble will collapse. The fact that the scale factor in the internal region increases exponentially over time, $a \sim \exp(Ht)$, is of no essential importance; it is simply a coordinate effect. It is important to know how the physical volume of the bubble increases: $V_{phys} = V_{coord}(t)\exp(3Ht)$. Since $V_{coord}(t)$ may tend toward zero, V_{phys} may do so also, despite the "inflationary" exponential function. This is what occurs during the collapse of the remnants of an old phase accompanied by the formation of black holes.[5] On the other hand, it is clear that if the region occupied by the fluctuation is larger than the internal Hubble radius $1/H$, then a shell moving at a velocity below the velocity of light cannot contract as long as the vacuum equation of state holds in the internal region (i.e., it cannot contract before the inflation time $\tau \sim 60/H$ at the very earliest). In this case, however, the physical size of the region occupied by the fluctuation is also large and may not fit into some arbitrary universe prepared for it. In such a situation, gravitational effects in the transition layer are extremely important and must be analyzed in detail.

To study the space-time structure of the resulting configuration, it is convenient to write the metric in terms of normal Gaussian coordinates

$$ds^2 = -dn^2 + e^{\nu} d\tau^2 - r^2(\tau, n)\, d\Omega^2 \tag{1}$$

which are related to the bubble shell in such a way that the equation of the shell is $n = 0$, and we have $\nu(\tau, n = 0) = 0$, so that at $n = 0$ the coordinate τ coincides with the proper time at this shell.

In a curved space-time, the vector normal to the $r = \text{const}$ surface may be either spacelike or timelike. In the former case we would have

$$\Delta \equiv g^{\alpha\beta} r_{,\alpha} r_{,\beta} < 0\,, \tag{2}$$

and the corresponding region of space-time would be called the "R region" (in a flat space-time, the R region would span the entire manifold). In the latter case we would have

$$\Delta > 0. \tag{3}$$

This is a "T region."

The basic equation for a study of the motion of the bubble wall is[5]

$$\sigma_{in}\sqrt{\dot{\rho}^2 + 1 - \rho^2/a_{in}^2} - \sigma_{out}\sqrt{\dot{\rho}^2 - \Delta} = 4\pi\kappa\rho S_0^0\,, \tag{4}$$

where $\rho \equiv r(\tau, n = 0)$, $a_{\text{in}}^{-2} = 8\pi\kappa\epsilon_{\text{in}} r^2/3$, ϵ_{in} is the energy density inside the bubble, $\sigma \equiv \text{sign}(\partial r/\partial n)$, κ is the gravitational constant, and S_0^0 is the surface energy density at the shell. In R regions we also have $\sigma = \text{sign}(\partial r/\partial q)$ for any spatial coordinate q whose value increases with distance from the center of the bubble.

We now prove the following assertion:

Assertion. A shell may inflate only in a T region of infinite expansion or in an R_- region of the external metric.

Proof. A shell cannot inflate in a T_- region, since the physical volume of the region occupied by the fluctuation decreases in this case. Furthermore, the size of the inflating fluctuation will eventually exceed the internal Hubble radius,

$$-\Delta_{in} = 1 - \frac{\rho^2}{a_{in}^2} < 0\,, \tag{5}$$

which means that the shell will intersect the T region of the internal metric. From Eq. (4) and the conditions $S_0^0 > 0$ and $\sigma_{\text{out}} > 0$ we then find $\Delta_{\text{out}} > 0$. This result means that a shell cannot be simultaneously in an R_+ region of the external metric and a T region of the internal metric.

If the boundary of the fluctuation moves in the outer R_- region, then either the overall configuration is equivalent to a mole's hole (this is possible in either a closed or

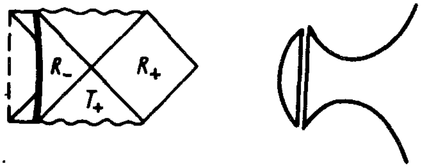

FIG. 1.

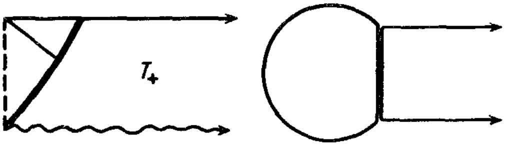

FIG. 2.

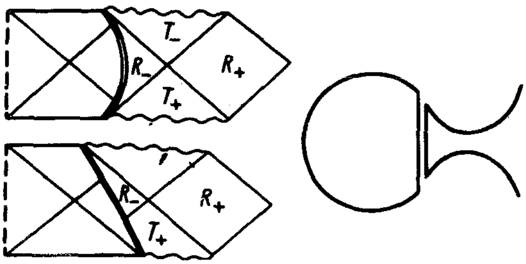

FIG. 3.

FIGS. 1–3. Global geometry of a space-time containing an inflationary region. Shown schematically at the right of each of the Penrose diagrams is the corresponding spatial section at a certain time. For simplicity, only the near R_+ regions to the right of the shell are shown here (the shell is shown by the double line in all cases).

open external universe) or the fluctuation occupies more than half of a closed universe (Figs. 1 and 3).

If the external region near the shell is a T region of unbounded inflation, then with motion (of light rays, for example) along it in the direction of increasing radial coordinate either an R_- region will be encountered, or there are no R regions at all in this metric. This fact (whose proof we omit) is apparently related to known theorems stating that the energy is positive, for otherwise an observer in an R_+ region would see energy arise from nothing.

The essence of the matter can be seen in the following simple discussion. We assume that we are dealing with a fluctuation of dimension r and energy density ϵ. The fluctuation must not be below its own gravitational radius (except in the case in Fig. 2); i.e., we have $r > r_g = 2\kappa m$. For the mass of the same fluctuation we have (discarding the kinetic energy) $m > (4\pi/3)\epsilon r^3 > 32\pi\kappa\epsilon m^3/3$. A necessary condition for inflation is thus the inequality $m^2 < 3M_{pl}^6/32\pi\epsilon$. On the other hand, the size of an inflating fluctuation is greater than the reciprocal of the Hubble radius,

$$a = \sqrt{\frac{3}{8\pi\epsilon}}\, M_{pl}$$

of the internal de Sitter metric, so that if the gravitational defect is not too large, we would have $m > (4\pi/3)\epsilon a^3$. We thus find $m^2 > (3/32\pi\epsilon)M_{pl}^6$. This contradiction means that the fluctuation forms a mole's hole (a semiclosed world, in which the gravitational effect of mass is important).

A study of the global geometry of the universe thus shows that a "fluctuation" in a randomly inflating universe may inflate only if it occupies more than half of a closed universe or if the global geometry of the universe corresponds to one of the cases shown in Figs. 1–3. In Figs. 1 and 3, the fluctuation forms a mole's hole, which represents a black hole for both the external universe (closed or open) and the internal universe. Such a black hole evaporates, causing the internal universe to split off from the external universe, thereby making the internal universe a closed world.

The mole's holes in Figs. 1 and 3 and the geometry in Fig. 2 require the specification of special boundary conditions even at the singularity. This is not an attractive result; on the contrary it would be quite reasonable to ask whether a mole's hole could form during the evolution of the universe.

We find the following scenario the most attractive one: An inflating universe is closed, and it is filled from the outset with a classical scalar field. Initial conditions of this sort might arise in a quantum origination of the universe (see Ref. 15 regarding quantum origination). Rubakov[16] makes a case for the possibility that a balance of the (negative) gravitational energy and the (positive) energy of excitations of matter may cause a tunnel transition to give rise to a relatively hot universe. We can assume that the appearance of the universe in an excited state is a quite general property of tunnel transitions in quantum gravitation, so that when it begins the universe is in fact *completely* filled by a smooth scalar field.

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