Horizontal unification of quarks and leptons: $SU(3)_H \otimes U(1)_H$ symmetry

Dzh. L. Chkareuli
Institute of Physics, Academy of Sciences of the Georgian SSR, Tbilisi

(Submitted 20 March 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 11, 473–477 (10 June 1985)

A model for the local horizontal symmetry $SU(3)_H \otimes U(1)_H$ with a violation scale on the order of the Planckian mass M_P is discussed. This violation gives rise at the same time to a spontaneous violation of the *CP* invariance. The mixing angles of the quarks are calculated. The *CP* phase and the masses of the *d*, *s*, and *b* quarks are in good agreement with the experiment. The mass of the *t* quark is predicted to be $m_t = 37$ GeV. The hierarchy of the quark and lepton masses in terms of the generations can be explained with the help of the symmetry of the horizontal hypercharge $U(1)_H$.

1. It is reasonable to assume that the splitting of quarks according to generations and the mixing of quarks in a weak current correspond to a spontaneous violation of the horizontal $SU(3)_H$ symmetry between the quark-lepton generations.[1–3] It is possible that the scale of the exact, local $SU(3)_H$ symmetry, V_H, is even higher than the scale of the SU(5) symmetry,[4] V ($V \simeq 10^{15}$ GeV) and that the horizontal unification of quarks and leptons occurs only at distances in the Planck range, $V_H = 0(M_P)$. We will adopt here this hypothesis[5] and discuss its principal consequences.

 0021-3640/85/110577-05$01.00

2. According to this hypothesis ($V_H \gg V$), in the $SU(3)_H$ limit we have an actual $SU(5) \otimes SU(3)_H$ symmetry with a chiral filling of fermions in its left-handed basis multiplet [SU(5) indices are omitted everywhere in this letter],

$$(\bar{5} + 10)_L^\alpha, \qquad \alpha, \beta, \ldots = 1, 2, 3\ (SU(3)_H), \tag{1}$$

in which the components of the quark and lepton fields at the right are assumed to be the triplets and the components of the quark and lepton fields at the left are assumed to be the antitriplets of $SU(3)_H$.

A spontaneous violation of SU(5) symmetry is caused by the standard scalars: 24-plet of Φ and 5-plet φ.[4] A violation of $SU(3)_H$ symmetry can be caused by a simple set of scalars—triplets $\eta_\alpha^{(m)}$ ($m = 1,2,\ldots$) and sextets $\chi^{(n)}_{\{\alpha\beta\}}$ ($n = 1,2,\ldots$) of $SU(3)_H$. At the same time, these scalars, along with the field φ, close to Yukawa couplings of quarks and leptons (1). Because of its simple scalar composition, the model rejects all but the renormalizable general couplings:

$$\mathcal{L}_Y = \frac{1}{M_P}\,[10^\alpha\, 10^\beta\, \varphi\, \chi^{(n)} F^{(n)} + \bar{5}^\alpha\, 10^\beta\, (\chi^{(n)}_{\{\alpha\beta\}} G^{(n)} + \eta^{(m)}_{[\alpha\beta]}\, \tilde{G}^{(n)})], \tag{2}$$

where $F^{(n)}$, $G^{(n)}$, and $\tilde{G}^{(m)}$ are the dimensionless coupling constants, and $\eta^{(m)}_{[\alpha\beta]} \equiv \epsilon_{\alpha\beta\gamma}\eta^{(m)\gamma}$. Since the scale for the violation of horizontal symmetry is $V_H = 0(M_P)$, we can logically assume that coupling (2) is induced by the gravitation (see also Ref. 6). If not all the couplings, at least those couplings that include the same multiplets (in terms of SU(5) and $SU(3)_H$ simultaneously) of the fermions and scalars must have the same constants. In other words, we assume that

$$F^{(n)} = F, \quad G^{(n)} = G \ (n = 1, 2, \ldots); \ \tilde{G}^{(m)} = G \ (m = 1, 2, \ldots). \tag{3}$$

The fermions in (1) and the scalars φ, χ, and η in couplings (2), in addition to being characterized in terms of the properties of SU(5) and $SU(3)_H$, are also characterized by the horizontal hypercharge $Y\,[U(1)_H]$ which will be discussed in Section 5.

The vacuum expectation values $\langle\chi^{(n)}\rangle$ and $\langle\eta^{(m)}\rangle$ of order $0(M_P)$ generate n $\chi^{(n)}_{\{\alpha\beta\}}$ sextets and m $\eta^{(m)}_{[\alpha\beta]}$ antitriplets and lead to a total spontaneous violation of $SU(3)_H \otimes U(1)_H$. After the scalar φ condenses, this violation also determines the mass matrices of the up quarks (n, c, t) and the down quarks (d, s, b) and leptons (e, μ, τ), respectively:

$$\hat{M}^u_{\alpha\beta} = F S_{\alpha\beta}, \quad S_{\alpha\beta} \equiv \frac{\langle\varphi\rangle}{M_P} \sum_n \langle\chi^{(n)}_{\{\alpha\beta\}}\rangle, \tag{4a}$$

$$\hat{M}^d_{\alpha\beta} = G S_{\alpha\beta} + \tilde{G} A_{\alpha\beta}, \quad A_{\alpha\beta} \equiv \frac{\langle\varphi\rangle}{M_P} \sum_m \langle\eta^{(m)}_{[\alpha\beta]}\rangle. \tag{4b}$$

3. As we can see from (4), the violation of $SU(3)_H$ symmetry leads at the same time to a spontaneous *CP* violation (the constants F, G, and $\tilde{G}$ are real) if the vacuum expectation values of the fields χ and η in the $\hat{M}^u$ and $\hat{M}^d$ matrices acquire a nontrivial relative phase. This phase is necessarily equal to $\pi/2$ if the Higgs potential $P(\chi,\eta)$ of the fields χ and η includes, in addition to the standard terms, the minimal couplings[1)]

$$h^{(m)}\eta^{(m)}_{[\alpha\beta]}\eta^{(m)}_{[\gamma\delta]}\chi^{(m)\{\alpha\gamma\}}\chi^{(m)\{\beta\delta\}}+\text{H.c.},\quad m=1,\ 2,\ \dots \tag{5}$$

It is easy to see that if the real constants $h^{(m)}$ are positive, it would be desirable in the case of the potential to set the complete phase of all vacuum expectation values in (5) equal to π, which means that (for all $\alpha\neq\beta$)

$$\arg\langle\eta^{(m)}_{[\alpha\beta]}\rangle-\arg\langle\chi^{(m)}_{\{\alpha\beta\}}\rangle=\pi/2,\ \arg\hat{M}^{d}_{\alpha\beta}=-\arg\hat{M}^{d}_{\beta\alpha}. \tag{6}$$

Let us define more accurately the structure of the mass matrices $\hat{M}^u$ and $\hat{M}^d$. For the set of horizontal vacuum expectation values ($n=1,2,3;m=1,2$)

$$\langle\chi^{(1)}_{\{\alpha\beta\}}\rangle\neq\langle\chi^{(1)}_{\{12\}}\rangle,\quad\langle\chi^{(2)}_{\{\alpha\beta\}}\rangle=\langle\chi^{(2)}_{\{23\}}\rangle,\quad\langle\chi^{(3)}_{\{\alpha\beta\}}\rangle=\langle\chi^{(3)}_{33}\rangle,$$

$$\langle\eta^{(1)}_{[\alpha\beta]}=\langle\eta^{(1)}_{[12]}\rangle,\quad\langle\eta^{(2)}_{[\alpha\beta]}\rangle=\langle\eta^{(2)}_{[12]}\rangle, \tag{7}$$

which can be determined through a natural selection of parameters in the polynomial P, the matrices $\hat{M}^u$ and $\hat{M}^d$ become the Fritzsch matrices.[7] It is assumed that the matrix elements have the following experimentally observable hierarchy:

$$|\hat{M}^u_{33}|:|\hat{M}^u_{23}|:|\hat{M}^u_{12}|=m_t:\sqrt{m_tm_c}:\sqrt{m_cm_u}\sim1:p:p^4, \tag{8a}$$

$$|\hat{M}^d_{33}|:|\hat{M}^d_{23}|:|\hat{M}^d_{12}|=m_b:\sqrt{m_bm_s}:\sqrt{m_sm_d}\sim1:p:p^3, \tag{8b}$$

where p is a numerical parameter that lies in the range 0.15–0.20. To insure that hierarchy (8) will hold, we must assume that

$$\langle\chi^{(3)}\rangle:\langle\chi^{(2)}\rangle:\langle\eta^{(2)}\rangle:\langle\eta^{(1)}\rangle:\langle\chi^{(1)}\rangle\sim1:p:p^2:p^3:p^4 \tag{9}$$

for the moduli of the vacuum expectation values in (7) [we recall that only the vacuum expectation values of the sextets $\chi^{(n)}=1,2,3;\langle\chi^{(3)}\rangle=0(M_P)$ contribute to $\hat{M}^u$]. As a result, the $\hat{M}^u$ and $\hat{M}^d$ matrices yield

$$s_1=\left|\sqrt{\frac{m_d}{m_s}}+e^{i\delta}\sqrt{\frac{m_u}{m_c}}\right|, \tag{10}$$

$$s_2=\frac{\sqrt{\frac{m_d}{m_s}}}{s_1}\left|\sqrt{\frac{m_c}{m_t}}-e^{-i\kappa}\sqrt{\frac{m_s}{m_b}}\right|,\quad s_3=\sqrt{\frac{m_um_s}{m_cm_d}}\,s_2$$

for the Kobayashi-Mascawa mixing angles of quarks ($s_k\equiv\sin\theta_k,k=1,2,3$).[7] Using phase conditions (6) and taking explicitly into account hierarchy (9) of the contributions of the triplets $\eta^{(1)}$, $\eta^{(2)}$, and sextets $\chi^{(1)}$, $\chi^{(2)}$ to the matrix elements $\hat{M}^d_{12}$ and $\hat{M}^d_{23}$ in (4b), we find

$$\delta=\frac{\pi}{2}\pm\arctan\left(p\frac{G}{\tilde{G}}\right),\qquad|\kappa|=\arctan\left(p\frac{\tilde{G}}{G}\right), \tag{11}$$

for the phase of the CP violation of $\eta\delta$ and for the phase κ. For $\tilde{G}\simeq G$ the values for the sines of the mixing angles s_1, s_2, and s_3 obtained from (11) [Eq. (16)] are in good agreement with the experiment of Ref. 8.

4. The mass matrix $\hat{M}^d$ in (4b) also accounts, by virtue of the SU(5) symmetry, for the mass ratios of the down quarks and leptons

$$m_d m_s = m_e m_\mu \,, \quad m_s m_b = m_\mu m_\tau \,, \quad m_b - m_s = m_\tau - m_\mu \,. \tag{12}$$

The first and third equations in (12) yield the following masses of the d, s, and b quarks,[2] consistent with experiment [if we take into account the renormalization of the quark masses from the SU(5) limit, $V = 10^{15}$ GeV, to 1 GeV for the s and d quarks and to 5 GeV for the b quark and if we assume, in accordance with the current algebra, that $m_s/m_d \simeq 20$]:

$$m_d \simeq 6{,}5\,\text{MeV}, \quad m_s \simeq 130\,\text{MeV}, \quad m_b \simeq 4{,}8\,\text{GeV} \tag{13}$$

The second equation in (12), which arises from the contribution to the matrix $\hat{M}^d$ in (4b) of the sextet $\chi^{(2)}$, sharply contradicts the experimental results, however. To "destroy" this equation without destroying the other two equations in (12), we must add to couplings (2) the coupling of the down quarks with a scalar 45-plet of the SU(5) symmetry, σ, which presumably has a horizontal hypercharge Y_σ that is different from the hypercharge Y_φ of the 5-plet, φ:

$$\frac{1}{M_P}\,\bar{5}^{\alpha}\,10^{\beta}\,\bar{\sigma}\,\lambda_{\{\alpha\beta\}}\,G', \qquad Y_\sigma = Y_\varphi + (Y_\lambda - Y_{\chi^{(2)}}), \tag{14}$$

where λ is a new fourth sextet which generates a vacuum expectation value "parallel" to the vacuum expectation value of the sextet, i.e., by means of the components $\langle\lambda_{\{23\}}\rangle \sim \langle\chi^{(2)}_{\{23\}}\rangle$; Y_λ and $Y_{\chi^{(2)}}$ are the hypercharges of the fields λ and $\chi^{(2)}$, and G' is a coupling constant ($G' \sim G$).

Separating in the matrix element $\hat{M}^d_{23}$ the contributions of the sextets $\chi^{(2)}$ from those of λ (the contribution of the triplet $\eta^{(2)}$ to $|\hat{M}^d_{23}|$ is negligible, on the order of p^2 as compared with the main contribution), we find from (4a) and (4b) one more mass formula

$$\frac{|\hat{M}^d_{23}(\chi^{(2)})|}{|\hat{M}^d_{33}|} = \frac{|\hat{M}^u_{23}|}{|\hat{M}^u_{33}|}\,, \qquad 3\sqrt{\frac{m_s}{m_b}} + \sqrt{\frac{m_\mu}{m_\tau}} = 4\sqrt{\frac{m_c}{m}}\,. \tag{15}$$

Using m_s and m_b in (13) for the physical mass of the t quark, from (15) we find $m_t = 37$ GeV. From the masses of the d, s, b, and t quarks in (13) and (15), along with the mass of the c quark $m_c = 1.35$ GeV, we finally find mixing angles (10) and δ phase (for $\tilde{G}\simeq G$ and $p = 0.17$).

$$s_1 = 0{,}22, \quad s_2 \simeq 0{,}04, \quad s_3 \simeq 0{,}01, \quad \sin\delta \simeq 0{,}985. \tag{16}$$

5. We attribute the hierarchy of the quark masses based on the generations [Eqs. (8a) and (8b)] and the difference between the masses of the up and down quarks to the symmetry of the horizontal hypercharge $\text{U}(1)_H$. In addition to the fields χ, η, and λ, we introduce the field I and the singlet $SU(3)_H$. We will now choose the hypercharges of these fields and of the fermion multiplets $\bar{5}^\alpha$ and 10^α in such a way that the following conditions will hold:

$$2Y_{10} + Y_\varphi + Y_{\chi^{(3)}} = 0, \qquad Y_{\bar{5}} + Y_{10} - Y_\varphi + Y_{\chi^{(3)}} + Y_I = 0, \tag{17a}$$

$$Y_{\chi^{(3)}} = Y_{\chi^{(2)}} + Y_I = Y_{\eta^{(2)}} + 2Y_I = Y_{\eta^{(1)}} + 3Y_I = Y_{\chi^{(1)}} + 4Y_I. \tag{17b}$$

The new Yakawa couplings [instead of couplings (2) and (14)], along with the fields χ, η, and λ, now contain, because of the $U(1)_H$ symmetry (17), a particular number of fields I with the appropriate powers $1/M_P$. According to hypercharges (17b), the matrix $\hat{M}^u$ acquires only the element $\hat{M}^u_{33}$ in the order $1/M_P$, the element $\hat{M}^u_{23}$ in the order $1/M_P^2$ and the element $\hat{M}^u_{12}$ in the order $1/M_P^5$, yielding hierarchy (8a) with the parameter $p = \langle I \rangle / M_P$, where $\langle I \rangle$ is the vacuum expectation value of the field I. Working in a similar way, we obtain the hierarchy of the elements (with the same parameter p) in the $\hat{M}^d$ matrix of (8b). The Yukawa couplings of the down quarks will have, by virtue of (17a), a superfluous power $1/M_P$ (in comparison with the couplings of the up quarks), which accounts for their relatively small masses. Because of the $U(1)_H$ symmetry, a special hierarchy of the vacuum expectation values (9) is thus no longer needed; all horizontal scalars χ, η, λ, and I with hypercharges (17) presumably generate roughly equal vacuum expectation values , $V_H \approx pM_P$, and have the same Yukawa constants.

I am deeply indebted to Z. G. Berezhiani, O. V. Kancheli, and K. A. Ter-Martirosyan for useful discussions.

[1)]Couplings of the type in (5) arise under any circumstances in the polynomial P in higher orders of Yukawa couplings (2).

[1]Dzh. L. Chkareuli, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 684 (1980) [JETP Lett. **32**, 671 (1980)].
[2]Z. G. Berezhiani and Dzh. L. Chkareuli, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 494 (1982) [JETP Lett. **35**, 612 (1982)]; Usp. Fiz. Nauk **145**, 165 (1985) [Sov. Phys. Uspekhi **28**, 104 (1985).
[3]F. Wilczek, Preprint NSF-ITP-83-08, 1983.
[4]P. Langacker, Phys. Rep. **72**, 187 (1981).
[5]Dzh. L. Chkareuli, A paper presented at the U.S.-Soviet working meeting, Erevan, 1983.
[6]J. Ellis and M. K. Gaillard, Phys. Lett. **88B**, 315 (1979).
[7]H. Fritzsch, Nucl. Phys. **B155**, 189 (1979).
[8]D. V. Nanopoulos, Preprint CERN-TH, 3995/84.

Translated by S. J. Amoretty