# Magnetic mass of the gluon in the presence of quarks

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The exact spinor structure of the Green's function of quarks in $QCD_4$ is found for $T \neq 0$. A nonperturbative expression is found for the quark-gluon vertex in the limit of a zero gluon momentum. The presence of quarks does not alter the infrared limit of the gluon propagator in $QCD_4$. This limit is the same as that given by the standard expression in quarkless gluodynamics.

An urgent problem in gluodynamics today is to study the thermodynamic and kinetic properties of the quark subsystem of $QCD_4$. Because of the singular properties of the effective quark-gluon interaction, the system of quarks in $QCD_4$ undergoes a phase transition at $T, \mu \neq 0$, for which we have only a phenomenological description at present. Furthermore, at $T \neq 0$ the spectrum of elementary excitations of the quark system has several qualitatively new features (a splitting of the spectrum, a minimum on one of the branches, etc.), which are not part of quantum field theory. The spinor structure of the Green's function of the quark at $T \neq 0$ is far richer. Its nontrivial stature in statistics stems from the breaking of Lorentz invariance and the appearance of a new vector $u_\mu$, associated with the heat reservoir. This fact was first noted by Fradkin[1] in quantum electrodynamics for the photon propagator, but the exact spinor structure of the Fermi propagator at $T \neq 0$ has not been determined previously in either quantum electrodynamics or quantum chromodynamics (QCD). This complication of the spinor structure of the Fermi propagator has been mentioned by Klimov[2] for massless quarks, but that study does not fully cover the general case.

The spinor structure of the quark Green's function of $QCD_4$ depends on the gauge and takes its simplest form in the axial gauge $A_4 = 0$, where the gauge vector $n_\mu$ is identically equal to the vector characterizing the heat reservoir, $u_\mu$. However, the expressions that have been derived go beyond the case of the axial gauge, also applying in certain other gauges, e.g., the Feynman gauge. An important point is that there be no special vectors other than the two vectors $p_\mu$ and $u_\mu$, which determine the function $G$. The quark Green's function is a bispinor, so that it should be representable as an expansion in the complete system of the five generators $(1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_5, \gamma_5\gamma_\mu)$. The structure functions of this expansion are constructed from the two vectors $p_\mu$ and $u_\mu$, which determine $G$. The Euclidean metric is chosen everywhere; $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$; $\gamma_\mu^2 = 1$ and we use the standard definition of the matrix $\sigma_{\mu\nu}$. The most general form of the bispinor $G^{-1}$ for the $A_4 = 0$ gauge,

$$G^{-1} = i(\hat{p}\, b_1 + \hat{u}\, b_2) + m + \frac{d}{2}(\hat{u}\hat{p} - \hat{p}\hat{u}), \tag{1}$$

is determined by four scalar functions that depend separately on $|\mathbf{p}|$ and $p_4$. For the $A_4 = 0$ gauge, there is neither a pseudoscalar nor a pseudovector in expression (1), since the latter entities cannot be constructed from the two vectors $p_\mu$ and $u_\mu$. When

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we switch to other gauges, there are changes in the scalar functions that determine (1): changes in the functions themselves and also in their number. The quark Green's function found with the help of (1) is

$$G = \frac{-i(\hat{p}b_1 + \hat{u}b_2) + m - \frac{d}{2}(\hat{u}\hat{p} - \hat{p}\hat{u})}{m^2 + (p_\mu b_1 + u_\mu b_2)^2 + d^2(p^2 - (up)^2)}, \tag{2}$$

where the dispersion relation

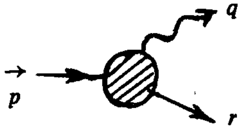
$$m^2 + (p_\mu b_1 + u_\mu b_2)^2 + d^2(p^2 - (up)^2) = 0 \tag{3}$$

determines (with the help of four functions) two different spin modes, each having one or several branches. A spectrum of this sort was found by Klimov[2] for the simplest case of a massless theory, in which the functions $m^2$ and $d^2$ are identically zero because of the $\gamma_5$ invariance in (3). The qualitative difference between the quark Green's function at $T \neq 0$ and that of field theory stems from the existence of the vector of the medium, $u_\mu$, in complete analogy with the change in a medium in the tensor structure of the polarization operator, found previously.[1] The iteration of function (2) in the rest frame of the medium ($u_i = 0$, $u_4 = 1$) begins with the standard expression

$$G_0 = \frac{-i\hat{p} + \gamma_4\mu_0 + m_0}{(p_4 + i\mu_0)^2 + \mathbf{p}^2 + m_0^2} \tag{4}$$

and has standard perturbative expansions of the functions $b_i$ and $m$ in powers of $g^2$, aside from the function $d$, which begins with a term of order $g^4$.

The quark-gluon vertex in quantum chromodynamics,

$\Gamma_{\psi\bar{\psi}V}(p, r\,|\,q)^{ija}_{\mu}$ =  , (5)



can also be expanded for $T \neq 0$ in a complete system of five generators, by analogy with (1):

$$\Gamma_{\psi\bar{\psi}V}(p, r\,|\,q)^{ija}_{\mu} = ig\left(\frac{\lambda^a}{2}\right)^i_j \Gamma_\mu(p, r\,|\,q),$$

$$\Gamma_\mu(p, r\,|\,q) = A_\mu \cdot 1 + B_{\mu\nu}\gamma_\nu + C_{\mu\nu\gamma}\sigma_{\nu\gamma} + \gamma_5(Z_\mu + \gamma_\nu E_{\mu\nu}), \tag{6}$$

where none of the coefficients is generally zero. It is assumed in (6) that the quarks transform in accordance with a fundamental representation of the SU(3) color group [$\lambda/2$ are its generators]. Expression (6) simplifies only for a massless theory, in which two of the five coefficients remain nonzero ($B_{\mu\nu} \neq 0$ and $E_{\mu\nu} \neq 0$). The choice of a gauge does not simplify (6) or change its tensor structure, although it does have a substantial effect on the choice of the coefficients that determine expression (6). The axial gauge is a special case, for which we have the simple Ward identities (which were independently derived in Ref. 3).

$$q_\mu \Gamma_{\psi\bar\psi V}(p,r|q)^{ija}_\mu = g\left(\frac{\lambda^a}{2}\right)^i_j [G^{-1}(p) - G^{-1}(r)] \qquad (7)$$

and simplified expressions for the coefficients in (6) in $n_\mu = u_\mu$. For the $A_4 = 0$ gauge, for example, the tensor form of the pseudovector $Z_\mu$ and of the pseudotensor $E_{\mu\nu}$ can easily be determined:

$$Z_\mu = z\epsilon_{\mu\nu\rho\lambda}(p+r)_\nu (p-r)_\rho u_\lambda,$$

$$E_{\mu\nu} = \epsilon_{\mu\nu\rho\lambda}[e_1(p+r)_\rho (p-r)_\lambda + e_2(p+r)_\rho u_\lambda + e_3(p-r)_\rho u_\lambda]. \qquad (8)$$

The scalar function $e_2(|\mathbf{p}|, p_4)$ in (8) is identically zero if we require that (7) hold. The other coefficient functions determining (6) have a more complicated tensor structure, but they can be reconstructed unambiguously.

For the axial gauge in $QCD_4$ we have the following simple formula by virtue of the simple Ward identities in (7):

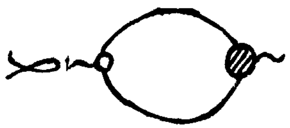
$$\Gamma_{\psi\bar\psi V}(p,p|0)^{ija}_n = \left(g\frac{\lambda^a}{2}\right)^i_j \frac{\partial G^{-1}(p)}{\partial p_n}. \qquad (9)$$

This formula can be used to calculate the infrared limit from the momentum transfer $q_\mu(q_4 = 0, |q| \to 0)$ of the spatial components $(n = 1,2,3)$ of the vertex function if we know the quark Green's function

$$G^{-1}(p) = (i\hat{p} - \gamma_4\mu_0 + m_0) + \Sigma(p). \qquad (10)$$

In contrast with quantum electrodynamics, for which the expression analogous to (9) holds in any gauge, in the theory of non-Abelian gauge fields this expression is valid only in the axial gauge, in particular, in the $A_4 = 0$ gauge, which is preferable for constructing various nonperturbative calculation methods. In the present letter we use (9) to show that the infrared limit of the polarization operator of the gluons in $QCD_4$ is determined exclusively by the gluomagnetic interaction and is not altered by the presence of quarks.

The infrared limit of the polarization operator of gluons in $QCD_4$ for the $A_4 = 4$ gauge is determined by, in addition to the usual diagrams of quarkless chromodynamics,[4] the quark loop

$\Delta m^2_{\text{mag}}$  , (11)

which must be calculated with the help of the exact quark Green's function, (2), and the exact quark-gluon vertex, (5). For the latter, however, we need only the infrared limit in order to calculate the loop in (11), so that its contribution to the magnetic mass of the gluon can be calculated exactly with the help of (9):

$$\Delta m^2_{\text{mag}} \propto \frac{g^2}{\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \mathrm{Sp}\left(\gamma_n G(p) \frac{\partial G^{-1}(p)}{\partial p_n} G(p)\right)$$

$$= - \frac{g^2}{\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \mathrm{Sp}\left(\gamma_n \frac{\partial G(p)}{\partial p_n}\right) = - \frac{g^2}{\beta} \sum_{p_4} \int \frac{d^3p}{(2\pi)^3} \frac{\partial}{\partial p_n}(p_n \widetilde{G}(p)) \qquad (12)$$

The sum over $n(n = 1,2,3)$ in (12) has not yet been calculated, and the function, $\widetilde{G}(p)$ is found from expression (2) after the trace of the $\gamma$ matrices is calculated in (12). The further transformations of expression (12) involve an integration by parts and the assertion that the "surface term" which arises here is zero, i.e.,

$$\lim_{|\mathbf{p}|} \frac{1}{\beta} \sum_{p_4} \left. |\mathbf{p}|^3 \widetilde{G}(|\mathbf{p}|, p_4) \right|_0^\infty = 0. \qquad (13)$$

Accordingly, there is no change in the magnetic mass of the gluon when quark loop (11) is taken into account. This mass is given by the same expression as has been found for quarkless gluodynamics[4]:

$$m^2_{\text{mag}} = \kappa^2 g^4 T^2 \ln(\Lambda / g^2 T), \qquad (14)$$

where $\Lambda \sim T$ for $g^2 \ll 1$.

This result, $(\Delta m^2_{\text{mag}})^Q = 0$, is important for both $QCD_4$ and quantum electrodynamics, where this fact has always been assumed[1] but has not been proved. The qualitative features of the behavior of the effective interaction between quarks at both small and large momentum transfer result exclusively from the gluomagnetic interaction according to the equation $(\Delta m^2_{\text{mag}})^Q = 0$, which we have proved. With regard to certain questions, it is therefore possible to treat quarks as constituting an independent subsystem of Fermi particles in an invariable potential. The structure found for the Green's function of the quark makes it possible to study the spectrum of elementary excitations of the quark subsystem, which has a nontrivial structure for $T$, $\mu \neq 0$, in contrast with field theory. In statistics, because of the breaking of Lorentz invariance, this spectrum has at least two branches with qualitatively different dispersion relations. Klimov[2] has studied this spectrum previously in the single-loop approximation for high temperatures, $T \gg \mu$, $m$, but it is important to use (3) to find this spectrum for a more general case. This analysis of the spinor structure of the quark-gluon vertex will be useful in the development of nonperturbative calculations, where it is necessary to determine the exact form of the quark-gluon vertex, which goes beyond the scope of Eq. (9), and to find the relationship between the scalar functions that determine it and the structure functions of the single-particle quark Green's function.

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