Restoration of the gauge invariance in γ_5 anomalous theories through the introduction of local counterterms

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Introduction of scalar chiral fields in the γ_5 anomalous gauge theories restores the gauge invariance of the theory through the addition of local counterterms.

We know that the existence of γ_5 anomalies[1] in the gauge theories leads to a violation of the Ward identities, i.e., to the loss of gauge invariance at the quantum level. It is generally assumed, therefore, that the theories with γ_5 anomalies are internally inconsistent. In view of this fact, can a gauge invariance in the theories with γ_5 anomalies be restored? A trivial way of accomplishing this task is to introduce additional fermions into the theory in such a way that there would be a total cancellation of the γ_5 anomalies. The gauge invariance can be restored also by introducing into the Lagrangian a nonlocal gauge-noninvariant counterterm which restores the gauge invariance at the quantum level.[2] Green and Schwarz[3] have recently shown that the anomalies in the supersymmetry gauge theory in the 10-dimensional space-time can be canceled out in the case of gauge groups SO(32) and $E_8 \times E_8$ by introducing the local counterterms.

In this letter we show that the introduction of scalar chiral fields in the γ_5 anomalous theories allows the gauge invariance of the theory to be restored through the introduction of the local counterterms.

Let us examine a model which describes the interaction of the Abelian gauge field with the fermion and scalar fields with the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}_L(i\hat{\partial} + e\hat{A})\psi_L + \bar{\psi}_R i\hat{\partial}\psi_R$$
$$+ \bar{\chi}_L i\hat{\partial}\chi_L + \bar{\chi}_R(i\hat{\partial} + e\hat{A})\chi_R + |\partial_\mu\varphi - ieA_\mu\varphi|^2$$
$$- h\bar{\chi}_R\chi_L\varphi - h\bar{\chi}_L\chi_R\varphi^+ - \lambda|\varphi^+\varphi - c^2| \, . \tag{1}$$

Lagrangian (1) is invariant under transformations

$$(\psi_L, \chi_R, \varphi) \to \exp(ie\alpha(x))(\psi_L, \chi_R, \varphi), \tag{2}$$
$$A_\mu \to A_\mu + \partial_\mu\alpha, \quad \chi_L \to \chi_L, \quad \psi_R \to \psi_R .$$

Model (1) is free of the γ_5 anomalies. A spontaneous violation of the symmetry in the field χ gives rise to a mass $M_\chi = hc$. The field $\varphi(x)$ can be represented in the form $\varphi(x) = |\varphi(x)|\exp(i\rho/x)$. Let us carry out a gauge transformation of (2) with the parameter $\alpha = -\rho$. Lagrangian (1) can be written in the form

$$\mathcal{L} = \bar{\psi}_L (i\hat{\partial} + e\hat{A} - e\hat{\partial}\rho)\psi_L + \bar{\chi}_R (i\hat{\partial} + e\hat{A} - e\hat{\partial}\rho)\chi_R - h\bar{\chi}\chi|\varphi| + \dots . \quad (3)$$

According to the Appelquist-Carazzone[4] theorem, the effect of a fermion on the low-energy spectrum can be ignored if the fermion mass is large $\chi(h \to \infty)$ [this is obvious in the case of Lagrangian (3) in the perturbation theory]. As a result, we find an effective Lagrangian

$$\mathcal{L}' = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}_L (i\hat{\partial} + e\hat{A} - e\hat{\partial}\rho)\psi_L . \quad (4)$$

Lagrangian (4) is invariant under transformations

$$\begin{aligned} \psi_L &\to \psi_L , \\ A_\mu &\to A_\mu + \partial_\mu \alpha , \\ \rho(x) &\to \rho(x) + \alpha(x). \end{aligned} \quad (5)$$

The presence of a γ_5 anomaly causes the fermion determinant (in contrast with the pure vector interaction) to be dependent on the field $\rho(x)$ in a nontrivial manner; specifically,

$$\mathcal{L}_{eff}(A_\mu, \rho) = \mathcal{L}_{eff}(A_\mu, \rho = 0) - \frac{e^3}{24\pi^2} \rho \epsilon^{\mu\nu\alpha\beta} \partial_\mu A_\alpha \partial_\nu A_\beta . \quad (6)$$

Note that a gauge-invariant field $B_\mu = A_\mu - \partial_\mu \rho$ describes a vector field with spin 1 plus a scalar field. Because of the anomalous nonconservation of the current, the Lorentz condition $\partial^\mu B_\mu = 0$ does not hold, and the scalar field $\varphi = (1/\Box)\partial^\mu B_\mu$ interacts in a nontrivial manner with the transverse part of the field B_μ. The Lagrangian which describes the interactions of the field φ and the vector field $B_{\mu\perp}$ is

$$\mathcal{L} = -\frac{e^3}{24\pi^2} \varphi \epsilon^{\mu\nu\alpha\beta} \partial_\mu B_\nu \partial_\alpha B_\beta . \quad (7)$$

Unfortunately, the theory described by Lagrangian (4) is not a renormalizable theory. We should point out that the fermion determinant $\mathrm{Det}[(i\hat{\partial} + e\hat{A})(1 + \gamma_5/2)]$ is invariant under gauge transformations (1) for the gauge fields that satisfy the condition $F\tilde{F} = 0$.

Since the generalization of these arguments to the non-Abelian case is straightforward, we will write only the basic equations.

Let us examine a Lagrangian of the type

$$\mathcal{L}' = -\frac{1}{2} \mathrm{Tr} F_{\mu\nu}^2 + \bar{\psi}_L (i\hat{\partial} + \hat{B})\psi_L ,$$

$$B_\mu = U^+ A_\mu U + \frac{1}{i} \partial_\mu U^+ U, \quad U^+ U = I. \quad (8)$$

Since the field B_μ does not change under gauge transformations of the type

$$U \to g U,$$
$$A_\mu \to g A_\mu g^{-1} + \frac{1}{i} \partial_\mu g g^{-1} \quad , \tag{9}$$

symmetry (9) is valid at the quantum level. The γ_5 anomaly manifests itself in that the fermion determinant becomes dependent in a nontrivial manner on the chiral field U(x). Under infinitesimally small gauge transformations a change in the effective action is given by[1]

$$\delta_\epsilon \Gamma(B) = -\frac{1}{24\pi^2} \int d^4x \, \epsilon^{k\lambda\mu\nu} \operatorname{Tr}[\epsilon(\partial_k B_\lambda + \partial_\mu B_\nu + \frac{i}{2} \partial_k (B_\lambda B_\mu B_\nu))] . \tag{10}$$

Using (10), we find

$$\Gamma(B) = \Gamma(A) + \Delta\Gamma(B; U) ,$$

$$\Delta\Gamma(B, U) = -\int_0^1 dt \frac{1}{24\pi^2} \epsilon^{k\lambda\mu\nu} \operatorname{Tr}[\omega(\partial_k B_\lambda^t \partial_\mu B_\nu^t + \frac{i}{2} \partial_k (B_\lambda^t B_\mu^t B_\nu^t))] d^4x , \tag{11}$$

$$B_\mu^+ = U_+^+ A_\mu U_t + \frac{1}{i} \partial_\mu U_+^+ U_t ,$$
$$U(x, t) = \exp[t \ln U] = \exp(wt). \tag{12}$$

Action (11) is the Wess-Zumino action.[5] In contrast with the Abelian case, $\Delta\Gamma(B,\mathrm{U})$ is nonlocal in the non-Abelian case. Lagrangian (8), however, is local. To restore gauge invariance, it is thus sufficient to introduce a local counterterm of the type

$$\Delta \mathcal{L} = \mathcal{L}'(B) - \mathcal{L}'(A). \tag{13}$$

The method for restoring the gauge invariance discussed above works for an arbitrary number of space-time dimensionalities and can be used to restore the gauge invariance in Kaluza-Klein models.

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