Higher-order derivatives in the interaction

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It is shown on the basis of a Lie model that an interaction with higher-order derivatives does not necessarily lead to physically unacceptable consequences.

A novel situation has arisen in a recent development of the theory of higher-order spins[1]: the presence of higher-order, and leading, derivatives in the interaction. This result immediately raises several questions, involving the formulation of a Cauchy problem, canonical quantization, and renormalizability. [Previously, if higher-order derivatives did appear in a theory, e.g., in the course of a regularization of divergences, they were concentrated in a kinetic part of the action, a part quadratic in the fields, which has traditionally determined the formulation of the Chauchy problem, canonical quantization, and the set of states (the composition of the particles).] It appears to us, however, that the basic danger here is the appearance of new states (not determined by the kinetic part of the action) with an indefinite metric. Our purpose in this letter is to clarify this point.

The basic hypothesis can be stated as follows: It is clearly possible that in weak fields an interaction can be treated as a perturbation of a main kinetic term (which, as before, determines the composition of the particles), and a standard perturbation theory with ordinary propagators may be possible.

This hypothesis is based on the following arguments. We treat theories with higher-order derivatives in the interaction as a limiting case of the ordinary theories, adding corresponding terms with higher-order derivatives to the kinetic part of the action with vanishing small coefficients. In this manner we can simultaneously avoid the difficulty of canonical quantization. In this process we find additional states with an indefinite metric, but with large, unbounded "seed" masses. The hope is that the interaction will not make these masses finite, so that in the limit the additional states will not contribute to physical processes (Green's functions).

The difficulties of nonrenormalizability, which are quite probable in such theories, should not stop us at the outset: With the experience we have gained so far, we can hope that these difficulties can be overcome by means of mechanisms that are unknown at this point (although we cannot, of course, rule out the possibility that nonrenormalizability and the existence of new states with an indefinite metric are inherently related).

As an example that verifies this possibility, we consider here a modification of the standard Lie model.[2]

We consider the action

$$S = \int d^4x\, L, \qquad L = \Delta L_{(\mathrm{kin})} + L_{(\mathrm{Lie})} + \Delta L_{(\mathrm{in})} . \tag{1}$$

The term $L_{(\mathrm{Lie})}$ describes the following version of the Lie model:

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$$L_{(\mathrm{Lie})} = V^+(i\partial_t - m_V)V + \sum_{k=1}^{n} C_k^{-1} N_k^+ (i\partial_t - m_k)N_k + \Theta^+(i\partial_t - \omega)\Theta + \sum_{k=1}^{n} g(V^+ N_k \Theta + N_k^+ V \Theta^+), \quad (2)$$

where m_V and m_k are independent of the momentum, and $\sum_{k=1}^{n} C_k m_k^s = 0$ $(s = 1,...n-1)$. The term

$$\Delta L_{(\mathrm{in})} = -g_1 \sum_{k=1}^{n} (V^+ N_k \ddot{\Theta} + N_k^+ V \ddot{\Theta}^+)$$

or, equivalently,

$$\Delta L_{(\mathrm{in})} = g_1 \sum_{k=1}^{n} [(\dot{V}^+ N_k + V^+ \dot{N}_k)\dot{\Theta} + (\dot{N}_k^+ V + N_k^+ \dot{V})\dot{\Theta}^+],$$

describes an interaction with higher-order derivatives. According to the discussion above, the additional term

$$\Delta L_{(\mathrm{kin})} = -\alpha V^+(i\partial_t - m_V)^2 V - \beta \sum_{k=1}^{n} C_k^{-1} N_k^+(i\partial_t - m_k)^2 N_k - \gamma\Theta^+(i\partial_t - \omega)^2\Theta$$

serves an auxiliary and intermediate role. We must examine the limit $\alpha, \beta, \gamma \to 0$.

A distinction between this model and the ordinary Lie model (a distinction in addition to the presence of an interaction with higher-order derivatives, of course) is that one N-particle is replaced by a set of N_k particles, $k = 1,...,n$, with a generally indefinite metric. The existence of an indefinite metric here should not be disturbing, since we wish only to demonstrate that the presence of higher-order derivatives in the interaction does not lead to *new* states with an indefinite metric. The purpose of this modification is to make the model regular.

A canonical quantization of this model leads to ordinary Feynman rules. Like the original Lie model, this model can be solved exactly. The solution reduces essentially to the single-loop result for the mass operator of the V particle. The N_k and θ particles, like the vertices, do not grow. The Green's function of the V particle is

$$S_V = \langle V V^+ \rangle_0, \quad S_V^{-1} = S_{0V}^{-1} + i\Pi.$$

The free Green's function

$$S_{0V}^{-1} = i[\alpha(E - m_V)^2 - (E - m_V)], \quad p = (E, \mathbf{p}),$$

has two poles, one corresponding to an ordinary particle of mass $\sim m_V$, and the other corresponding to an additional, heavy particle (of mass $\sim 1/\alpha$), with an indefinite metric. The mass operator is

$$\Pi(p) = -\frac{1}{(2\pi)^3}\int d\mathbf{l} \sum_{k=1}^{n} C_k \Big\{ [g + g_1\omega^2(\mathbf{l})]^2 \times$$

$$\times \left[\frac{1}{\omega(\mathbf{l}) + m_k - E - i\epsilon} - \frac{1}{\omega(\mathbf{l}) + m_k + \frac{1}{\beta} - E - i\epsilon} \right] - \left[g + g_1\left(\omega(\mathbf{l}) + \frac{1}{\gamma}\right)^2 \right]^2$$

$$\times \left[\frac{1}{\omega(\mathbf{l}) + m_k + \frac{1}{\gamma} - E - i\epsilon} - \frac{1}{\omega(\mathbf{l}) + m_k + \frac{1}{\beta} + \frac{1}{\gamma} - E - i\epsilon} \right] \Big\}.$$

If we make the usual assumption $\omega(\mathbf{l}) \sim |\mathbf{l}|$ as $|\mathbf{l}| \to \infty$, then it is sufficient to set $n = 7$ to find a limit on $\Pi(p)$ as β, $\gamma \to 0$ and, correspondingly, a limit on S_V as α, β, $\gamma \to 0$:

$$\lim_{\alpha,\beta,\gamma \to 0} S_V^{-1}(p) = -i\Big\{(E - m_V)$$

$$+ \frac{1}{(2\pi)^3}\int d\mathbf{l} \sum_{k=1}^{n} C_k [g + g_1\omega^2(\mathbf{l})]^2 \frac{1}{\omega(\mathbf{l}) + m_k - E - i\epsilon}\Big\}.$$

If we have $m_V < (\min_k m_k) + (m_\theta = \omega(0))$, as usual, then for moderately small values of g and g_1 we have one stable V particle, and no new states appear (!). The limit can be taken in such a way that the second pole of S_{0V}, corresponding to the additional heavy V particle with an indefinite metric, goes onto the second sheet when the interaction is taken into account; i.e., this heavy particle becomes a decay particle. Correspondingly, the prelimit theory has only a single stable V particle and a unitary S matrix with a good limit. (The only nontrivial point is N-θ scattering, whose amplitude reduces to the propagator S_V within obvious factors.) The unitarity of the limiting theory can, of course, also be tested here directly without any difficulty, but it is still useful to know that the final theory is a limit of unitary theories: This fact may prove to be of general importance and may prove useful in less trivial situations.

In summary, this model is a precise example of the possibility under discussion here: The introduction of higher-order derivatives in the interaction does not change the theory in any qualitative way.

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[1]F. A. Berends, G. J. H. Burgers, and H. van Dam, Z. Phys. C24, 247 (1984).
[2]S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, Row, Peterson, & Co., Evanston, Illinois, 1961 (Russ. transl. IIL, Moscow, 1963).

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