Nonlinear effect of an alternating magnetic field on the dynamics of a domain wall in a ferromagnet

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It is predicted that the initial mobility of a domain wall can be restored by the action of an alternating magnetic field. This effect is attributed to the locking of spontaneous oscillations of the spins in a moving domain wall in the region where its velocity reaches a maximum.

The action of a variable driving force on a nonlinear system which undergoes spontaneous oscillations leads, as we know, to phenomena such as interference mixing of waves, forced mode locking, randomization, etc. These phenomena have been studied most extensively in weakly coupled superconducting systems and more recently in one-dimensional conductors in which space-charge waves are generated (see Refs. 1–3, for example). We will show that a moving domain wall in a ferromagnet exhibits analogous nonlinear properties and in this respect is a magnetic analog of these systems.[1–3] The dynamics of the domain wall in a static magnetic field H is characterized by the presence of a critical field H_c, above which the self-similar nature of the motion of the wall changes to a self-oscillating nature. In a Bloch domain wall the critical field is traceable to Walker's mechanism that restricts the velocity and in a twisted domain wall (a film) the critical field stems from the generation of horizontal Bloch lines.[4] If the magnetizing field is aligned in the plane of the film or if the basis anisotropy is adequate (as in the case of orthorhombic magnetic materials, for example), the twisted domain walls disappear and the limiting velocity increases. In fields higher than the critical field, the mobility of a domain wall decreases sharply under any condition because of spontaneous oscillation of the spins in the moving domain wall.[5] The forced synchronization of spontaneous-oscillation frequency restores, under the influence of an alternating magnetic field, the differential mobility of the domain wall to its initial value, which shows that the domain-wall mobility is self-oscillating in nature. This phenomenon is similar to the harmonic current steps that appear on the I-V characteristic of a Josephson point contact which is subjected to a microwave bombardment—the Shapiro effect.[6]

Let us consider this effect using as an example a high-Q, uniaxial, orthorhombic ferromagnet, in which $K_u \gg K_\perp \gg 2\pi M^2$, where K_u and $K_\perp$ are the energies of the uniaxial and basis anisotropies, and M is the magnetization. The anisotropy parameters proposed here allow us to use the averaged Landau-Lifshitz equations, which were used by Slonzewski,[4,7] to describe the dynamics of a domain wall:

$$\dot{q} M / \gamma \Delta = K_\perp \sin 2\psi + \dot{\psi} \alpha M/\gamma , \tag{1}$$

$$\dot{q} = (\Delta\gamma/\alpha)(H - \dot{\psi}/\gamma) + (\Delta\gamma/\alpha) H_\sim \cos \omega t, \tag{2}$$

where q is the coordinate of the center of the domain wall, ψ is the angle at which the magnetization emerges from the wall, γ is the gyromagnetic ratio, Δ is the wall thickness, α is the attenuation constant in the Landau-Lifshitz equations, $H_{\sim}$ is the amplitude of the alternating displacive magnetic field, and ω is the frequency of this field. We can infer from (1) and (2) that

$$2\omega_k^{-1}\dot{\psi} + \sin 2\psi = H/H_c + (H_{\sim}/H_c)\cos\omega t, \qquad (3)$$

where $\omega_k = 2K_{\perp}\alpha\gamma/1+\alpha^2$, and $H_c = K_{\perp}\alpha/M$. Equation (3) is similar to the equation that describes the evolution of an abrupt change in the phase at a superconducting point contact.[1] This equation has been studied in detail and has been solved both numerically and analytically.[2] Equations (1) and (2) were solved by Slonzewski for the dynamics of a domain wall at[7] $H_{\sim} = 0$. In the latter case ($H_{\sim} = 0$), Eq. (3) implies that at $H < H_c$ we would have $\psi = \arcsin(h/H_c)$. We can therefore find from (1) the velocity $V = \mu_0 H$, where $\mu_0 = \Delta\gamma/\alpha$ is the initial mobility. At $H > H_c$ the solution of (3) for $H_{\sim} = 0$ is oscillatory in nature, whose dependence of the spontaneous-oscillation frequency on the magnetic field is described by the equation $\omega_A = \omega_k\sqrt{H^2/H_c^2 - 1}$. The translational velocity, which can be found by averaging Eq. (2) over the oscillation period, is $V = \langle\dot{q}\rangle = \mu_0(H - \langle\dot{\psi}\rangle/\gamma) = \mu_0[H - \sqrt{H^2 - H_c^2}/(1+\alpha^2)]$.At $H > H_c$, there is a sharp decrease in the velocity and differential mobility $\mu = dV/dH = \mu_0 \times [1 - H/\sqrt{H^2 - H_c^2}(1+\alpha^2)]$, since we typically have $\alpha \ll 1$.

In an alternating field whose amplitude is moderately large, $H_{\sim} \ll H$, and whose frequency ω is far from the spontaneous-oscillation frequency ω_S, the functional dependence $V(H)$ changes only very slightly (quadratic averaging). However, in the neighborhood of the points at which the phases match, near $H_\omega = H_c\sqrt{1+\omega^2/\omega_k^2}$ and $V_\omega = V_k[\sqrt{1+\omega^2/\omega_k^2} - \omega/\omega_k(1+\alpha^2)]$, where $V_k = \mu_0 H_c$, there is a nonlinear oscillation frequency locking, $\omega_S = \omega$ (forced mode locking) which causes the functional dependence $V(H)$ to change markedly. Let us mismatch the phase by the amount $\theta = 2\psi - \omega t - \pi/2$. Assuming that this quantity changes slower than the angle $\psi(t)$,

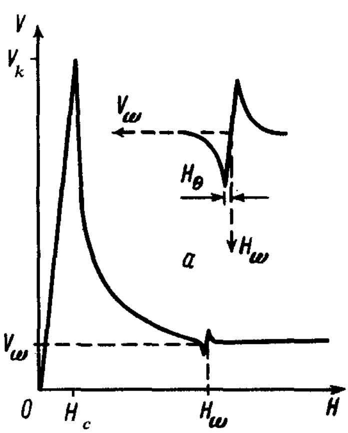

FIG. 1. The translational velocity of the domain wall versus the static magnetic field $V(H)$ with an alternating component of the displacive field $H_{\sim} \neq 0$. a—A magnified section of the $V(H)$ curve in the neighborhood of the point $H = H_\omega$, $V = V_\omega$.

by analogy with Ref. 1, for example, we can derive from (3) for the case of a small-amplitude perturbation, $H_{\sim} \ll H$, an evolution equation for $\theta(t)$ which has a structure similar to that of Eq. (3) at $H_{\sim} = 0$,

$$\omega_{\theta}^{-1} \dot{\theta} + \sin\theta = (H - H_{\omega}) / H_{\theta}, \tag{4}$$

where $\omega_\theta = \omega_k H_{\sim} / 2\sqrt{H^2 - H_c^2}$ and $H_\theta = H_c H_{\sim} / 2H$. This equation implies that the phase θ is constant in the field interval $|H - H_\omega| < H_\theta$, so that $\dot{\psi} = \omega/2 = \text{const}$. From (2) we find, therefore, that $\mu = d\langle\dot{q}\rangle/dH = \mu_0$ in this field interval; i.e., the initial mobility is restored. Outside of this interval, phase oscillations $\theta(t)$ occur at a frequency $\Omega = \omega_0\sqrt{(H - H_\omega)^2/H_\theta^2 - 1}$, causing the mobility to decrease. Calculations show that in this case we have

$$V = V_{\omega} + \mu_0 (H - H_{\omega}), \quad |H - H_{\omega}| \leqslant H_{\theta}, \tag{5}$$

$$V = V_{\omega} + \mu_0 \left[H - H_{\omega} - \frac{H\sqrt{(H - H_{\omega})^2 - H_{\theta}^2}}{(1 + \alpha^2)\sqrt{H^2 - H_c^2}} \right] \quad |H - H_{\omega}| > H_{\theta}. \tag{6}$$

At $|H - H_\omega| \gg H_\theta$ far from $H = H_\omega$ the mobility which we determined from (6) tends to return to its former value $\mu \to \mu_S$, as in the case of $H_{\sim} = 0$. Near the critical values $|H - H_\omega| = H_\theta$, the mobility is negative, i.e., $dV/dH < 0$, a situation which may cause a flexural instability of the domain wall. Figure 1 is a plot of the velocity of the wall as a function of the static magnetic field $V(H)$ at $H_{\sim} \neq 0$. The greatest change in $V(H)$ occurs near $H = H_\omega$ and $V = V_\omega$. Figure 1a shows a blown-up version of the $V(H)$ curve in the neighborhood of this point. Similar changes also occur near the points $H_\omega^{(m)} = H_c\sqrt{1 + (m\omega/\omega_k)^2}$ and $V_\omega^{(m)} = V_k[\sqrt{1 + (m\omega/\omega_k)^2} - m\omega/\omega_k(1 + \alpha^2)]$, where $m = 2, 3, 4 \ldots$. The amplitude of the anomalies decreases rapidly with increasing number m. In the high-frequency limit $\omega \gg \omega_k$, the oscillation locking intervals of $H_\theta^{(m)}$ are described by the Bessel functions $H_\theta^{(m)} = H_c J_m(H_{\sim}\omega_k/H_c\omega)$.

Here are some numerical estimates of the effect. The magnetic parameters typical of rare-earth garnet ferrites are $K_\perp/M = 100$ G, $\gamma = 2\times 10^7$ s^{-1} Oe^{-1}, and $\alpha = 0.1$. In this case $H_c = 10$ Oe, so that at a field $H = 20$ Oe the spontaneous-oscillation frequency is $\omega_S/2\pi = 100$ MHz. The field interval of the forced mode locking H_θ is 25% of the alternating-field amplitude. Consequently, at $H_{\sim} = 5$ Oe, for example, it is $H_\theta = 1$ Oe, which is an appreciable value.

[1]A. F. Volkov and F. Ya. Nad', Pis'ma Zh. Eksp. Teor. Fiz. **11**, 92 (1970) [JETP Lett. **11**, 56 (1970)].
[2]M. J. Renne and D. Polder, Rev. Phys., Appl. **9**, 25 (1974).
[3]A. H. MacDonald an M. P. Plishke, Phys. Rev. B **27**, 201 (1983).
[4]A. Malozemov and J. C. Slonzewski, Domennye stenki v materialakh s tsilindricheskimi magnitnymi domenami (Domain Walls in Materials with Magnetic Bubbles), Mir, Moscow, 1982.
[5]F. H. De Leeuw, IEEE Trans. Magn., MAG-9, 614 (1973); W. T. Stacy, A. B. Voermans, and H. Logmans, Appl. Phys. Lett. **29**, 817 (1976).
[6]S. Shapiro, A. R. Janus, and S. Holly, Rev. Mod. Phys. **36**, 223 (1964).
[7]J. C. Slonzewski, Intern. J. Magnetizm. **2**, 85 (1972).

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