Absence of a renormalization group in localization theory

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The assumption that there is a renormalization group in localization theory contradicts perturbation-theory results.

The hypothesis of the existence of a renormalization group[1] has significantly influenced the recent development of the theory of disordered metals. According to this hypothesis, the change in the conductivity of a sample upon a change in its dimensions depends on only the magnitude of the conductivity and the magnitude of the change in dimensions. Renormalization-group equations have been written for the conductivity. Working from natural assumptions regarding the form of the Gell-Mann-Low function, Abrahams *et al.*[1] predicted a localization in an arbitrarily weak potential in one and two dimensions, and they also predicted a power-law behavior of the kinetic coefficients near the mobility threshold in a space with dimensionality $d > 2$. The hypothesis that a renormalization group exists has been reconciled with the results of a direct summation of the first few orders of a perturbation theory[2,3] and

with the results of an analysis of σ models in a space with dimensionality $2+\epsilon$(Refs. 4–6).

These calculations, however, have employed a dimensional regularization procedure in order to form integrals that diverge logarithmically in a two-dimensional space. According to this procedure, the calculations are carried out in a space with dimensionality $d<2$, and then an analytic continuation is made in the dimensionality of the space to $d \geqslant 2$. This procedure works well in the theory of phase transitions in the calculation of universal quantities, which are independent of the cutoff method.[7]

If, on the other hand, we do not approach the question convinced of the existence of universal quantities, we need to test the dimensional regularization procedure. A recent calculation[8] on a Cayley tree showed that the critical behavior near the mobility threshold in a disordered system is by no means a power-law behavior, and there exist a minimum metallic conductivity in the metallic phase and a maximum dielectric constant in the dielectric regions. This result is at odds with the renormalization-group predictions. We therefore need to test the renormalization group by direct calculations in two dimensions, without appealing to any additional assumptions of any sort, e.g., regarding regularization of integrals.

Let us consider a two-dimensional system of disordered metal granules. In this model, a calculation of the density correlation function $K(r, r')$, which determines the electron kinetics, reduces to an evaluation of integrals over supermatrices[8]:

$$K(r, r') = -2\pi^2\nu^2 \int (Q_{13}^{12})_r (Q_{31}^{21})_{r'} \exp(-F[Q]) \prod_i dQ_i \tag{1}$$

where

$$F[Q] = -\sum_{i,j} J_{ij} \operatorname{STr}\{Q_i Q_j\} + 2i(\tilde{\omega} + i\delta) \sum_j \operatorname{STr}\{\Lambda Q_j\}; \quad \tilde{\omega} = \frac{\omega\pi\nu V}{8}. \tag{2}$$

The indices i, j, r, r' in Eqs. (1) and (2) specify the particular granules. The constants j_{ij} describe the interaction between granules which is caused by the possibility of a tunneling of electrons from one granule to another; ν is the state density at the Fermi surface; V is the volume of the granules; and ω is the external frequency. It is assumed that all the granules have the same volume and that the coupling constants J_{ij} depend on only the distance between granules i and j. The supermatrices Q and Λ and also a definition of the supertrace STr are given explicitly in Ref. 9.

The model described by Eqs. (1) and (2) is a lattice supermatrix σ model. The question of a short-range cutoff does not arise in this model. If Q varies slowly from one granule to another, the model in (1) and (2) obviously converts into a continuum supermatrix σ model.[6,9]

In our perturbation-theory calculations for large values of the coupling constants J_{ij} or for a large interaction radius r_0, we restrict the discussion to a system with a broken symmetry with respect to time reversal because of technical difficulties. For lattice spin models there exists a regular method[10] for constructing expansions that are applicable at large values of the coupling constants or at large interaction radii. In this model, an average spin is singled out in the mean-field approximation, and then an expansion is carried out in terms of deviations from this average spin. A similar analy-

sis can be carried out for the model in (1) and (2). The expectation value of the supermatrix Q in this model is exactly equal to supermatrix Λ for any values of the coupling constants J_{ij}. Singling out this expectation value, we can put the functional $F[Q]$ in (2) in the form

$$F[Q] = -\sum_{i,j} J_{ij}\,\mathrm{STr}\{(Q_i - \Lambda)(Q_j - \Lambda)\} - \alpha \sum_j \mathrm{STr}\{\Lambda Q_j\}\,, \tag{3}$$

where $\alpha = 2(J - i(\tilde{\omega} + i\delta))$.

As in Ref. 10, we can pursue the evaluation of the integral in (1) by carrying out an expansion in the interaction of the deviations from the expectation value [the first term in (3)]. The calculation method and the method for constructing the one-loop and two-loop approximations are completely analogous to those in Ref. 10. The corresponding diagrams of the one-loop and two-loop approximations are given in the same paper. The actual expansion parameter is the large diffusion coefficient D_0, which is expressed in terms of the interaction constant $J(k)$ in the Fourier representation as follows:

$$D_0 = -\frac{1}{2} J''(0)\,. \tag{4}$$

For a large interaction radius r_0, this expansion is also valid at small values of J (a sufficient condition is $D_0 \gg 1$). After some rather lengthy calculations, we find the following result for the density correlation function $K(k)$ in the momentum representation:

$$K(k) = \frac{(\pi\nu)^2}{4(Dk^2 - i\tilde{\omega})}\,, \tag{5}$$

where the diffusion coefficient D is

$$D = D_0(1 - \delta) - \frac{64\pi^2}{D_0}(1 - \gamma)\ln\frac{\tilde{\omega}_0}{\tilde{\omega}}\,, \tag{6}$$

$$\delta = \frac{c}{D_0}\int \frac{(J_x'(p))^2}{J - J(p)}\,\frac{d^2p}{(2\pi)^2}\,,$$

$$\gamma = \frac{2^9 c^2}{\pi J^2}\int (J_x'(p))^2 J(p)(J - J(p))\,d^2p, \quad c = \frac{1 - e^{-8\alpha}}{32\,\alpha^2}\,. \tag{7}$$

The frequency $\tilde{\omega}_0$ is on the order of J, and $J_x'(p)$ is the derivative of $J(p)$ with respect to the x component of the momentum p.

If we formally set δ and γ equal to zero, expression (6) for the diffusion coefficient D becomes the same as the corresponding result derived by dimensional regularization.[3,4,6] The final values of δ and γ are determined by the short-range contribution. These quantities depend on the structure of the lattice. For large values of r_0 the parameter γ may be on the order of unity, since J may be less than or approximately equal to unity, while D_0 remains large. However, even if γ is small, incorporating this parameter at $r_0 \gg 1$ does not go beyond the accuracy of this analysis, since the following

orders would introduce higher powers of r_0^{-1}.

The dependence of the coefficient of the logarithm in (6) on the structure of the lattice contradicts the hypothesis of the existence of a renormalization group,[1] since that hypothesis would allow this coefficient to depend only on D_0. This hypothesis is therefore incorrect. The case of a broken symmetry with respect to time reversal is apparently not a special case, and the hypothesis of a renormalization group is also incorrect in other cases.

In the absence of a renormalization group, there is no basis for the assertions regarding localization in an arbitrarily weak potential at $d = 2$ and regarding a power-law behavior near the mobility threshold at $d > 2$. It would be more logical to have a mobility threshold at $d \geqslant 2$ and a minimum metallic conductivity, as predicted by Mott,[10] at all $d \geqslant 2$. The latter possibility is at least consistent with the results of an exact analysis on a Cayley tree. This calculation will be set forth in more detail in a separate paper.[12]

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