Dependence of the sticking of muons to helium in the muon-catalysis reactions $dt\mu \to \mu^4\text{He} + n$ and $dd\mu \to \mu^3\text{He} + n$ on the density of the mixture $D_2 + T_2$

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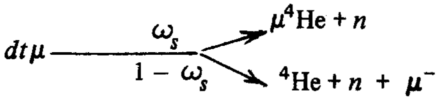
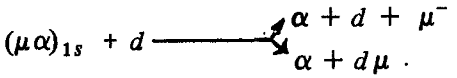
The probability for the sticking of muons to helium in the reactions $dt\mu \to \mu^4\text{He} + n$ and $dd\mu \to \mu^3\text{He} + n$ decreases with increasing density φ of the mixture of deuterium and tritium as a result of stepwise ionization of a mesic atom of helium during the collisions of muons with the *d* and *t* nuclei in the course of stopping.

1. In the experiments carried out by the Idaho-Los Alamos group[1] the sticking coefficient ω_s in the reactions of muon catalysis



$$dt\mu \xrightarrow[1-\omega_s]{\omega_s} \begin{cases} \mu^4\text{He} + n \\ {}^4\text{He} + n + \mu^- \end{cases} + \ 17.6\ \text{MeV} \qquad (1)$$

was found unexpectedly to depend on the density $\varphi = N/N_0$ of the mixture $D_2 + T_2$ ($N_0 = 4.25 \times 10^{22}\ \text{cm}^{-3}$ is the density of liquid hydrogen). This sticking coefficient cannot be explained in terms of the theoretical predictions,[2,3] in which it was assumed that during the stopping of a $(\mu^4\text{He})^+$ ion (below we call it $\mu\alpha$) with an initial energy $E_0 = 3.5$ MeV at which it was produced in reaction (1), the initial sticking coefficient ω_s^0 decreases to $\omega_s = 0.77\ \omega_s^0$ due to a charge exchange and direct ionization of $\mu\alpha$ from the 1*s* state:



$$(\mu\alpha)_{1s} + d \longrightarrow \begin{cases} \alpha + d + \mu^- & \text{(2a)} \\ \alpha + d\mu\, . & \text{(2b)} \end{cases}$$

(It was also assumed that a similar situation occurs upon the collision of $\mu\alpha$ with tritium nuclei *t*.)

2. We show that at $\varphi \sim 1$ a stepwise ionization of a mesic atom $\mu\alpha$

$$(\mu\alpha)_{1s} + d \to (\mu\alpha)_n + d \to \alpha + d + \mu^- , \qquad (3)$$

which was initially discussed in Ref. 3, becomes appreciable (see also Ref. 4).

A scheme for transition (3) is shown in Fig. 1. Also shown here are the rates λ_{1n}, λ_{in}, and $\lambda_{nn'}$ of the excitation $(\mu\alpha)_{1s} \to (\mu\alpha)_n$, of the ionization $(\mu\alpha)_n \to \alpha + \mu^-$, of the de-excitation $(\mu\alpha)_n \to (\mu\alpha)_{n'}$ (at $n > n'$), and of the excitation $n \to n'$ (at $n < n'$). Since the initial velocity $v_0 \approx 6$ a.u. at which the mesic atom $\mu\alpha$ is produced in reaction (1) is high, the λ_{1n} rates were calculated in the Born approximation of the linear trajectories. The λ_{in} and $\lambda_{nn'}$ rates were found in the Thomson limit, which is valid[5] at $n \geqslant 2$. The

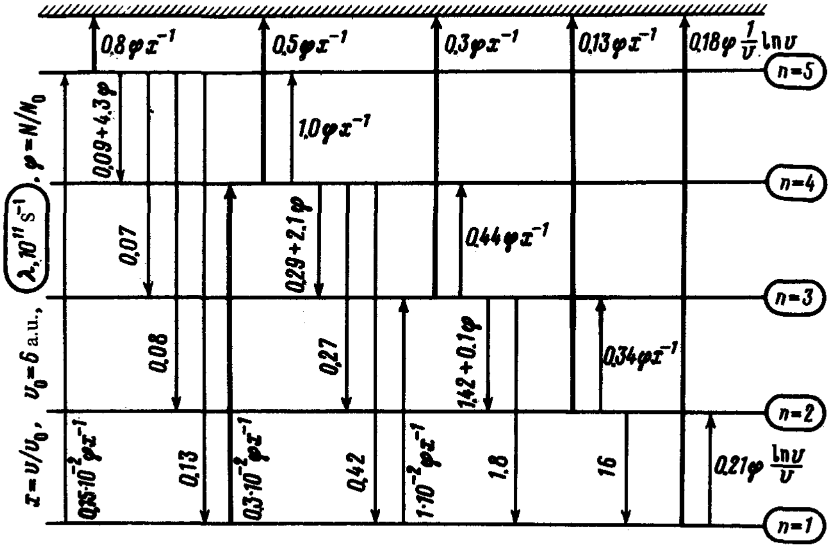


FIG. 1. A scheme for the transitions of a mesic atom $\mu\alpha$. All the velocities are given in units of $10^{11}\ \mathrm{s}^{-1}$; $\varphi = N/N_0$, $x = v/v_0$, $v_0 = 6$ a.u., and $N_0 = 4.25\times 10^{22}\ \mathrm{cm}^{-3}$.

$\lambda_{nn'}$ rates are determined by the φ-independent radiative transitions, by the Auger processes (at $n > n'$) and by the excitation induced by collisions (at $n < n'$) proportional to φ.

If the kinetic energy of $\mu\alpha$ is greater than ~10 eV, the Stark mixing rates $nl \rightarrow nl'$ of the $(\mu\alpha)_n$ states are extremely high $(\sim 10^{11} n^4 \varphi x^{-1} s^{-1}, x = v/v_0)$.[6] In Fig. 1 we have therefore averaged the $\lambda_{nn'}$ rates over l.

3. From Fig. 1 we see that the time evolution of the population densities N_n of the various states of $(\mu\alpha)_n$ and of the population density of the continuum N_c is consistent with the equations

$$\frac{dN_1}{dt} = -(\lambda_{i1} + \lambda_1)N_1 + \sum_{n'} \lambda_{1n'} N_{n'} \tag{4}$$

$$\frac{dN_n}{dt} = -(\lambda_{in} + \lambda_n)N_n + \sum_{n' > n} \lambda_{n'n} N_{n'} + \lambda_{1n} N_1 ,$$

$$\frac{dN_c}{dt} = \sum_n \lambda_{in} N_n + \lambda_{i1} N_1 ,$$

where $N_1(0) = 1$, $N_n(0) = N_c(0) = 0$, $n = 2,3,\ldots$, and $\lambda_n = \Sigma_{n'} \lambda_{nn'}$.

Since the $\lambda_{nn'}$ rates of the radiative and Auger transitions are considerably higher than the ionization λ_{in} and excitation λ_{1n} rates, processes (3) reach the quasi-steady-state regime, for which $dN_n/dt \approx 0$, in a time $\sim \lambda_{nn'}^{-1} \approx 10^{-12}$ s, and N_n can be determined from the algebraic system of linear equations

$$-(\lambda_{in}+\lambda_n)N_n + \sum_{n'>n}\lambda_{n'n}N_{n'} + \lambda_{1n}N_1 = 0. \tag{5}$$

We thus find $N_n = (\lambda_{1n}/\tilde{\lambda}_n)N_1 \equiv \tilde{N}_n$, where $\tilde{\lambda}_n$ are the effective de-excitation rates.

Since $\Sigma_n N_n/N_1 \ll 1$, and since the normalization conditions is $N_1 + N_c + \Sigma_n N_n = 1$, we find that the $(\mu\alpha)_{1s}$ state "decays" in accordance with

$$\frac{dN_1}{dt} = -\Lambda_1 N_1,$$

$$\Lambda_1 = \lambda_{i1} + \sum_n \lambda_{in}\tilde{N}_n, \tag{6}$$

which means that

$$N_1(\infty) = \exp\{-p_1 - p_2\}, \tag{7}$$

where

$$p_1 = \int_0^\infty \lambda_{i1}\,dt = 0,26 \text{ (Refs. 2 and 3)}$$

$$p_2 = \sum_n \int_0^\infty \lambda_{in}\tilde{N}_n\,dt. \tag{8}$$

The λ_{1n} and λ_{in} rates and the effective population densities $\tilde{N}_n$ depend in an approximate way (at $v \gg 1$ a.u.) on the velocity v of the mesic atom $\mu\alpha$ and on the density φ in the following way[5]:

$$\lambda_{1n} \sim \varphi n^{-3} v^{-1} \ln v, \quad \lambda_{in} \sim \varphi n^2 v^{-1}. \tag{9}$$

It follows, therefore, that $\tilde{N}_n \sim \varphi v^{-1}$. Figure 2 shows the $\tilde{N}_n$ curves for $\varphi = 1$ and two values of the velocity, $v = v_0 = 6$ and $v = 3$, as well as the effective ionization rates $\lambda_{in}\tilde{N}_n$.

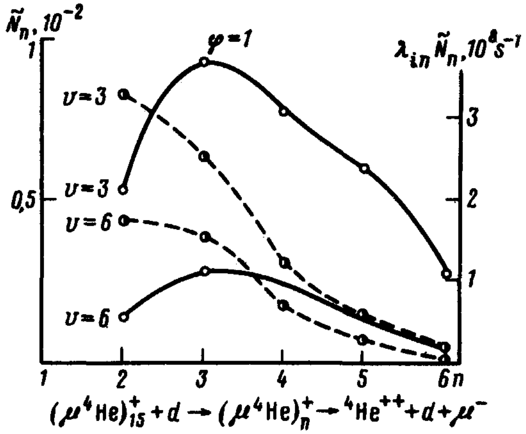


FIG. 2. Effective population densities $\tilde{N}_n$ and effective ionization rates $\lambda_{in}\tilde{N}_n$ of the excited states of $(\mu\alpha)_n$ at $\varphi = 1$, $v = 3$, and $v = 6$.

4. Using the Bethe-Bloch equation for stopping losses

$$\frac{dv}{dt} \approx - \frac{8\pi N_0 \varphi}{m_e M v} \ln v, \tag{10}$$

where M is the mass of $\mu\alpha$, from (8) we find that $p_2 = \varphi \int_0^{v_0} f(v) dv$, where the function $f(v) \approx 0.01$ a.u. and is nearly independent of v. For the reaction (1) we have $v_0 = 6$ and $p_2 \approx 0.06\varphi$, and for the reaction $dd\mu \rightarrow \mu^3\mathrm{He} + n$, we have $v_0 = 3.2$ and $p_2 = 0.03\varphi$. In other words, the stepwise ionization (3) reduces the values of the initial sticking coefficients ω_s^0 and ω_d^0 by 6 and 3%, respectively, during the time it takes $\mu\alpha$ and μ^3He to stop (0.6×10^{-10} s and $\sim 10^{-11}$ s, respectively, at $\varphi \approx 1$).

We know that approximately 30% of the muons in reaction (1) are brought into the excited states of $\mu\alpha$ (Refs. 2 and 3) and that a fraction of them are shaken off due to stopping $(\mu\alpha)_n$, thereby escaping the $(\mu\alpha)_{1s}$ state. The probability for these processes was determined in Ref. 7. Taking into account all the processes mentioned above, we can represent the resulting sticking coefficients for reaction (1) and for $dd\mu \rightarrow \mu^3\mathrm{He} + n$ by the interpolation formulas

$$\begin{aligned} \omega_s &= \omega_s^0 \left(1 - \frac{0.08\,\varphi}{0.8 + \varphi}\right) \exp\{- 0.26 - 0.06\,\varphi\}, \\ \omega_d &= \omega_d^0 \left(1 - \frac{0.12\,\varphi}{0.3 + \varphi}\right) \exp\{- 0.05 - 0.03\,\varphi\}, \end{aligned} \tag{11}$$

where the values $\omega_s^0 = 0.848 \times 10^{-2}$ and $\omega_d^0 = 0.133$ were calculated in Refs. 8, 9, and 10, respectively. At $\varphi = 1$ we thus find $\omega_s = 0.58 \times 10^{-2}$ and $\omega_d = 0.11$.

5. The actual values of ω_s and ω_d are smaller than those given above, because the following processes were ignored in the calculations: the charge exchange due to the excited states

$$(\mu\alpha)_n + d \rightarrow \alpha + (d\mu)_{n'}, \tag{12}$$

the excitation

$$(\mu\alpha)_n + e \rightarrow (\mu\alpha)_{n'} + e' \tag{13}$$

due to the collisions of $(\mu\alpha)_n$ in the states $n \gtrsim 4$ with electrons, and the increase in the stopping time of $\mu\alpha$ due to a decrease in the equilibrium charge of $\mu\alpha$ in the medium.

All these processes increase the probability for the shaking off of the muons and decrease the values of ω_s and ω_d. These values can be determined with more accurate calculations.

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