# Direct calculation of the conductivity of films with magnetic impurities

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The leading logarithmic correction to the conductivity of a two-dimensional metal containing magnetic impurities is calculated. The result has a nonuniversal dependence on the method used to cut off the logarithmic integrals at short range.

Considerable progress has recently been achieved toward an understanding of the mechanisms by which quantum interference affects transport phenomena in conductors with a slight degree of disorder ($p_F l \gg 1$).[1–4] It has been shown that the interference gives rise to mutual effects between scatterers separated by distances significantly greater than the mean free path $l$. These mutual effects are described by means of an effective interaction of soft modes—diffusons and cooperons[3,4]—and lead to corrections to the conductivity $\sigma(\omega)$ which are singular in the frequency $\omega$. The lower critical dimensionality in this problem is $d = 2$, and at $d = 2$ the singular corrections are described by logarithmic integrals. In the leading order in $1/p_F l$, the logarithmic correction is determined by a cooperon. If the conductor contains magnetic impurities, or if it is immersed in an external magnetic field, the cooperons are suppressed, and the leading logarithmic correction arises in order[5] $(1/p_F l)^2$. In order to calculate this correction, it is necessary to correctly regularize the logarithmic integrals at short range.[2,6] Hikami[6] has found this correction through an analytic continuation from the dimensionality of the space: $d = 2 - \epsilon$, $\epsilon \to 0$. This cutoff method is based on the assumption that the result does not depend on the properties of the system at short range. Physically, the cutoff arises at distances $\sim l$ at which the diffusive propagation of particles gives way to a ballistic propagation.

In the present letter we take distances $\sim l$ into account in calculating a correction to the conductivity, and we determine the conditions under which there is a difference from the result derived in Ref. 6.

Gor'kov *et al.*[3] have described a method for calculating corrections to the conductivity of an electron gas. For simplicity, we consider the case of isotropic scatterers, as in Ref. 3, but in the present letter, for the case with magnetic impurities, the dashed lines on the diagrams correspond to the quantity

$$f_{\alpha\beta\gamma\delta} = \frac{\delta_{\alpha\beta}\delta_{\gamma\delta}}{m\tau_0} + \frac{\vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}}{3m\tau_s} = \frac{1}{2m\tau}\left(\delta_{\alpha\delta}\delta_{\beta\gamma} + \left(1 - \frac{4\tau}{3\tau_s}\right)\vec{\sigma}_{\alpha\delta}\vec{\sigma}_{\gamma\beta}\right), \tag{1}$$

where the parameters $\tau_0$ and $\tau_s$ are the scale times for electron scattering without and with spin flip, $\tau = (\tau_0^{-1} + \tau_s^{-1})^{-1}$ is the mean free time, and $\vec{\sigma}_{\alpha\beta} = (\sigma^x_{\alpha\beta}, \sigma^y_{\alpha\beta}, \sigma^z_{\alpha\beta})$ are the Pauli matrices.

  

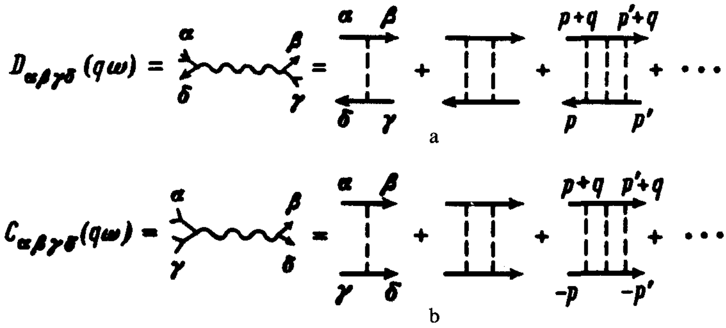


FIG. 1.

The propagators of the soft modes—diffusons $D(\mathbf{q}\omega)$ and cooperons $C(\mathbf{q}\omega)$—are described by the sum of the diagrams in Figs. 1a and 1b and have the following values:

$$D_{\alpha\beta\gamma\delta}(\mathbf{q}\,\omega) = \frac{1}{2m\tau}\left\{\frac{\delta_{\alpha\delta}\delta_{\beta\gamma}}{1-\pi} + \frac{(1-4\tau/3\tau_s)\vec{\sigma}_{\alpha\delta}\vec{\sigma}_{\gamma\beta}}{1-\pi(1-4\tau/3\tau_s)}\right\}, \tag{2}$$

$$C_{\alpha\beta\gamma\delta}(\mathbf{q}\,\omega) = \frac{(\delta_{\alpha\delta}\delta_{\beta\gamma} - \vec{\sigma}_{\alpha\delta}\vec{\sigma}_{\beta\gamma})\left(1-\left(1-\frac{2\tau}{\tau_s}\right)\left(1-\frac{2\tau}{3\tau_s}\right)\pi\right) - \frac{4\tau}{3\tau_s}\vec{\sigma}_{\alpha\delta}\vec{\sigma}_{\beta\gamma}}{2m\tau\left[1-\left(1-\frac{2\tau}{\tau_s}\right)\pi\right]\left[1-\left(1-\frac{2\tau}{3\tau_s}\right)\pi\right]},$$

where

$$\pi = \pi(\mathbf{q}\,\omega) = \frac{1}{m\tau}\int G^R_{\epsilon+\omega}(\mathbf{p}+\mathbf{q})G^A_\epsilon(\mathbf{p})\frac{d\mathbf{p}}{(2\pi)^2}$$

$$= \frac{1}{\sqrt{\mathbf{q}^2 l^2+1}} + i\omega\tau, \quad q \ll p_F, \quad \omega \ll \epsilon.$$

To calculate the corrections to the conductivity at $\omega\tau_s \ll 1$ which depend logarithmically on the frequency, we must calculate all the diagrams of order $(1/p_F l)^2\sigma_0$ that contain at least one diffuson. (For $\omega\tau_s \gg 1$, the presence of the magnetic impurities is inconsequential, and the problem reduces to that discussed in Ref. 3.) Diagrams of this sort are shown in Fig. 2. Among them we have some of order $\sigma_0(1/p_F l)^2 \ln^2(1/\omega\tau_s)$, but the terms $\sim \ln^2(1/\omega\tau_s)$ cancel out in the sum. The contribution that remains is proportional to $\sigma_0(1/p_F l)^2\ln(1/\omega\tau_s)$. To calculate the coefficient of the logarithm, we need to single out each logarithmic integration over the momentum of one of the diffusion lines and assume that this momentum and the frequency $\omega$ are zero in the other parts of the diagram. As a result, we find an expression for the coefficient of the logarithm in the form of an integral over the momenta of the remaining diffuson, cooperon, and electron lines. Although this calculation is carried out in the standard way, we should

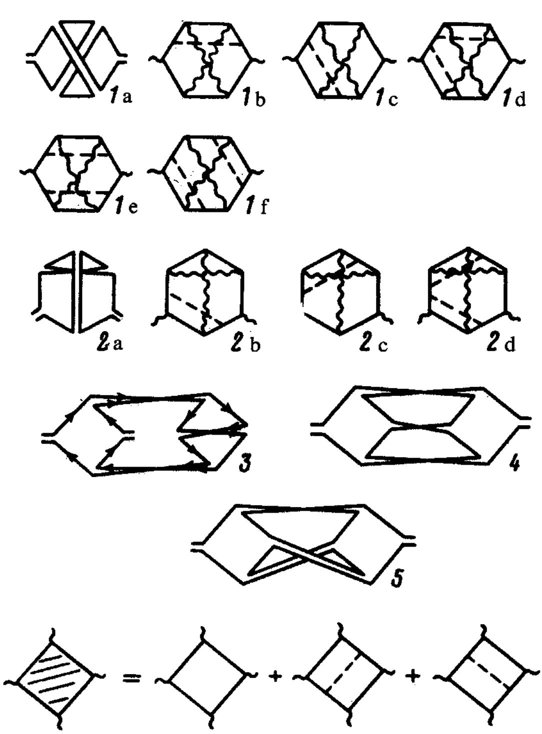


FIG. 2.

point out the following circumstance: All the terms in the series in Fig. 1 for the functions $D(\mathbf{q}\omega)$ and $C(\mathbf{q}\omega)$, except for the diagrams with one and two dashed lines, decay rapidly at $ql \gg 1$. Consequently, in evaluating the integrals over the momenta of the electron lines (the "tetragons" and "hexagons" in Fig. 2) we may assume that the momenta of the diffusons and cooperons are small in comparison with $p_F$ if the diffusons and cooperons contain more than two dashed lines. The contribution of low-order scattering processes should be treated separately. The analysis shows that all the diagrams which contain, in addition to one diffuson, either one or two dashed lines are not logarithmic in the sum, while in the diagrams containing more than two dashed lines the momenta of the diffusons and cooperons may be assumed small in comparison with $p_F$.

We thus find the following integral expression for the coefficient of $\ln(1/\omega\tau_s)$:

$$\frac{\delta\sigma}{\sigma_0} = -\left(\frac{1}{p_F l}\right)^2 \ln\frac{1}{\omega\tau_s} \int \frac{d\mathbf{q}}{(2\pi)^2}\left(K^d\left(\mathbf{q}, \frac{\tau}{\tau_s}\right) - K^c\left(\mathbf{q}, \frac{\tau}{\tau_s}\right)\right) = -\left(\frac{1}{p_F l}\right)^2 f\left(\frac{\tau}{\tau_s}\right) \ln\frac{1}{\omega\tau_s}, \tag{3}$$

where $K^d(\mathbf{q}, \tau/\tau_s)$, the contribution of diagrams 1a–1f and 4, is expressed in terms of the diffuson propagator $D(q)$, and $K^c(\mathbf{q}, \tau/\tau_s)$, i.e., the contribution of diagrams 2a–2d, 3, and 5, is expressed in terms of the cooperon propagator $C(\mathbf{q})$.

In the absence of magnetic impurities ($\tau_s \to \infty$) we have $D(\mathbf{q}) = C(\mathbf{q})$ and also $K^c(\mathbf{q}) = K^d(\mathbf{q})$, and the integrand in (3) cancels out. A cancellation of this type was first noted by Maleev and Toperverg,[4] but they did not examine effects that would disrupt the invariance under time reversal, and they did not ascribe these cancellations to this invariance.

In summary, the result depends on the ratio[1)] $\tau/\tau_s$. The coefficient of $\ln(1/\omega\tau_s)$ in (3) is the same as that found in Ref. 6 if the scattering with spin flip is weak, and the condition $\tau/\tau_s \ll 1$ holds. (At $\tau/\tau_s \ll 1$, a new distance scale arises: the scale length for diffusion without spin flip, $l_s \sim l\sqrt{\tau_s/\tau} \gg l$.) At shorter distances, at $ql_s \gg 1$, the diffusons and cooperons coincide: $D(\mathbf{q}, \tau/\tau_s) = C(\mathbf{q}, \tau/\tau_s) + O(\tau/\tau_s)$ and $K^d(\mathbf{q}) = K^c(\mathbf{q}) + O(\tau/\tau_s)$. The leading contribution to the integral in (3) comes from momenta $q \sim l_s^{-1} \ll l^{-1}$, which lie in the diffusion region. Singling out this contribution, we find $f(\tau/\tau_s) = 1/(2\pi)^2 + O(\tau/\tau_s)$.

In the opposite case, $\tau/\tau_s \sim 1$, the momenta $q \sim l^{-1}$ are important in integral (3). The calculation simplifies in the case $\tau/\tau_s = \frac{3}{4}$, in which an electron "forgets" the spin direction after each scattering [see (1)] and contributes $f \cong 3.12 \times 1/(2\pi)^2$.

This nonuniversal dependence of the coefficient in (3) on the properties of the conductor at short range shows that the hypothesis of single-parameter scaling suggested by Abrahams *et al.*[1] cannot be directly generalized to the case of a system with magnetic impurities, as has been done in Refs. 2, 6, and 8. Just which charges are correct for the equations of the renormalization group in this case remains an open question.

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[1)]A similar nonuniversality has been discovered independently by K. B. Efetov in a different model of a disordered system, which also lacks invariance under time reversal.[7]

[1]E. Abrahams, P. W. Anderson, D. C. Liccardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[2]F. Wegner, Z. Phys. **B35**, 207 (1979).
[3]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[4]S. V. Maleev and B. P. Toperverg, Zh. Eksp. Teor. Fiz. **69**, 1440 (1975) [Sov. Phys. JETP **42**, 734 (1975)].
[5]P. A. Lee, J. Noncryst. Solids **35**, 21 (1980).
[6]S. Hikami, Phys. Rev. B **24**, 2671 (1981).
[7]K. B. Efetov (in press).
[8]K. B. Efetov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **79**, 1120 (198) [Sov. Phys. JETP **52**, 568 (1980)].

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