Fluctuations in the extrinsic conductivity of disordered conductors

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The fluctuations in the static extrinsic conductivity of disordered metals from one sample to another are discussed. This conductivity of disordered metals is calculated on the basis of the Kubo equation. The mean-square deviation of the total conductivity of a cube from the expectation value of unity in units of $e^2/\hbar$ is found to be independent of the size of the cube and of its dimensionality. The difference between the observable conductivity and the conductivity found from the Kubo equation is calculated in the region of good metallic conductivity. The situation near the metal-insulator transition is discussed. The effect of spin scattering and of the magnetic field is considered.

The Kubo equation or an equivalent equation is always used to calculate the conductivity of metals. This means that the electric field in a sample is assumed to be uniform, that the current density of the sample can be calculated, and that its average is taken over the sample. The proportionality between the average current density and the field is determined by the specific conductivity of the sample which is denoted by $\sigma^{(K)}$.

The extrinsic conductivity $\sigma^{(ex)}$ measured experimentally can be determined from the ratio of the total current through the sample to the voltage applied to it. The conductivities $\sigma^{(K)}$ and $\sigma^{(ex)}$ differ from each other because the electric field inside a sample is nonuniform in any system with a static disorder. An analytic expression for $\sigma^{(ex)}$ can be found only in special cases even under highly simplified conditions which allow a local conductivity to be introduced.[1,2]

In this letter we discuss the large-scale inhomogeneities of a static conductivity, at $T = 0$, of a system of electrons in a random impurity potential which is assumed to be Gaussian and δ correlated. Instead of attempting to solve the complete system of equations in order to determine the partial current and field distributions in the given sample, we will formulate the problem in a different manner.

The conductivity $\sigma^{(K)}$ of the given sample of finite size L is a random quantity. Consequently, in addition to the conductivity $\langle\sigma^{(K)}\rangle$ which is averaged over the occurrences of the random potential and which is calculated in the usual manner, it would be justifiable to discuss the mean-square deviation

$$\langle\sigma^{(K)2}\rangle - \langle\sigma^{(K)}\rangle^2 \equiv \delta\sigma^2. \tag{1}$$

In the case of large L, the value of $\sigma^{(K)}$ fluctuates much more vigorously than the impurity concentration: $\delta\sigma^2 \sim (e^2/\hbar)^2 L^{4-2d}$ (d is the effective dimensionality of the sample which is determined the same way as that in the weak-localization theory[3]).

This result can be rewritten in terms of G—the total (rather than the specific) conductivity of the d-dimensional cube of size L:

$$\delta G^2 = \left(C_d \frac{2\sqrt{2}}{\pi^3} \frac{e^2}{\hbar}\right)^2 \equiv G_0^2, \tag{2}$$

where $C_1^2 = \pi^4/90$, $C_2^2 \simeq 1.51$, $C_3^2 \simeq 2.0$, and $\delta G^2 = \langle G^{(K)^2} \rangle - \langle G^{(K)} \rangle^2$. For an arbitrary dimensionality d the fluctuations $G^{(K)} = \sigma^{(K)} L^{d-2}$ in units of $e^2/\hbar$ are thus on the order of unity, regardless of the size of the sample, L. From (2) we see that the two-dimensional conductivity is not a self-averaging quantity even in the region of good metal.

If at $T = 0$ the sum of all the loops with two vector vertices must be calculated in order to determine $\langle \sigma^{(K)} \rangle$ by the impurity diagram technique,[4] then the quantity $\delta\sigma^2$ is determined by the sum of the four tails which are irreducible after the averaging. In calculating $\delta\sigma^2$ we should take into account that the irreducible diagrams have one fewer δ function of the incoming impulses than the reducible diagrams. This means that the diagrams for determining $\delta\sigma^2$ should be divided by the volume V of the system.

The sum of the diagrams (Fig. 1) which are characterized by the diffusion or Cooper poles (depending on the direction in which the arrows of the electron propagators are pointing) in the two-particle Green's function leads to relation (2). These poles are represented by the wavy lines in Fig. 1. Let us consider a sample with linear dimensions L_i, $i = 1,2,3$. If the incoming frequencies and the reciprocal of the relaxation time of the phase, τ_φ^{-1}, are small in comparison with D/L_i^2 (D is the diffusion coefficient of the electrons), then we would have

$$\delta\sigma^2 = \frac{8}{V}\left(\frac{\sigma}{\pi\nu}\right)^2 \sum_{\mathbf{q}} (Dq^2)^{-2}, \tag{3}$$

where ν is the electron density. From (3) and the quantization conditions[3]

$$q_i = \frac{\pi n_i}{L_i}\ ;\ n_3 = 1, 2, 3, \ldots\ ;\qquad n_1, n_2 = 0, 1, 2, 3, \ldots$$

we find Eq. (2) for δG^2.

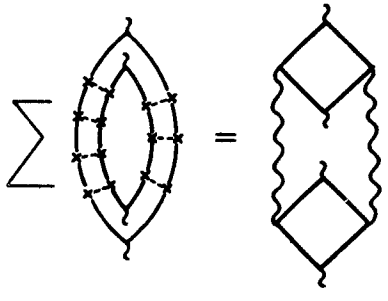

FIG. 1

Near the Anderson transition, the fluctuations are on the order of the conductivity. The agreement between $G^{(K)}e^2/\hbar$ and the scaling theory[5] implies that $L<\xi$ (ξ is the correlation length). From (2) we find that $\delta G^2/\langle G^{(K)}\rangle^2 \sim 1$ in the critical region and that $\langle G^{(K)}\rangle$ and $\langle G^{(ex)}\rangle$ differ from each other and vary from one sample to another by an amount of order $e^2/\hbar$, even if the scaling theory applies.

In a good metal we have $\delta G^2 \ll G^{(K)2}$. In the lower order in δG^2 we can calculate $G^{(ex)}$. We should point out that according to (2), the fluctuations of all scales are important. For finite fluctuations we have $\sigma^{(ex)} - \sigma^{(K)} \sim -\delta\sigma^2/\sigma^{(K)}$.[1] In our case we find

$$\frac{d\langle \sigma^{(ex)} - \sigma^{(K)}\rangle}{d\ln L} \propto -\frac{\delta\sigma^2}{\sigma^{(ex)}} = \frac{G_0^2}{\sigma^{(ex)}} L^{4-2d} \tag{4}$$

Specifically, in a two-dimensional sample we have

$$\langle G^{(ex)} - G^{(K)}\rangle \propto -\left(\frac{e^2}{\hbar}\right)^2 \frac{\ln L/l}{G^{(ex)}} \tag{5}$$

(l is the mean free path). Equation (5) is a good argument against the scaling theory of the Anderson transition: $G^{(ex)}$ and $G^{(K)}$ are the same only in the first logarithmic approximation. In the case of $G^{(ex)}$ and $G^{(K)}$, the indices and hence the correlation lengths must therefore be different.

At a finite temperature or a finite frequency ω and in the limit $L\to\infty$, we should substitute $L_\varphi = \sqrt{D\tau_\varphi}$ or $L_\omega = \sqrt{D/\omega}$ for L in (4) and (5).

The physical cause of the conductivity fluctuations of this sort seems to be linked to the correlations between the position of the exact energy levels in this example of the random potential.[6,7] The sensitivity of δG^2 to the spin scattering is evidence in favor of this argument. The electron spin can be changed through the scattering by magnetic impurities over a distance L_s and through the spin-orbit effects acting over the distance L_{so}, It turns out that

$$\delta G^2 = \alpha_s G_0^2. \tag{6}$$

Here $\alpha_s = \frac{1}{6}$ for $L_s < L$ and $\alpha_s = \frac{1}{3}$ in the range $L_{so} < L < L_s$. In the first case, only the diffusion contribution to δG^2 remains, the electron and the hole being in the singlet state. In the second case, the Cooper singlet contribution is also important.

The magnetic field H also changes δG^2, affecting at the same time the orbital motion of the electron and its spin. As a result, $\alpha_s = 1/2$ when $L_H < L < L_Z$. If $L_Z < L$, then $\alpha_s = 1/4$. [$L_H^2 = \hbar c/eH$, $L_Z^2 = D/(g\mu_B H)$, and $g\mu_B H$ is the Zeeman splitting.] The suppression of δG^2 by the magnetic field leads to an additional contribution to the magnetoresistance, which can be found by substituting L_H into (4) and (5). This contribution is negative and much smaller than the usual contribution[3] made under the condition $G \gg e^2/\hbar$. Near the transition, both contributions to the magnetoresistance are of the same order of magnitude.

Calculations of the dielectric-phase conductivity on a computer in Ref. 8 have

revealed the presence of a magnetoresistance whose sign depends on the quantity that is being averaged: In the case of the averaging of G^γ the magnetoresistance was negative for $\gamma < 1$ and positive for $\gamma \geqslant 1$. The part of the magnetoresistance in a metal attributable to the suppression of δG^2 by the magnetic field has these properties.

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