Kaluza-Klein theories and the space-time signature

I. Ya. Aref'eva and I. V. Volovich
V. A. Steklov Institute of Mathematics, Academy of Sciences of the USSR

(Submitted 11 May 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 12, 535–537 (25 June 1985)

Vacuum solutions in Kaluza-Klein theories are constructed with additional compactified time dimensions, for which the zeroth-order modes do not contain ghosts. Compact spaces of negative curvature are used.

Significant progress has recently been made toward the derivation of a unified theory of elementary particles in the Kaluza-Klein approach in supergravity[1] and superstrings.[2,3] It is assumed in this approach that the space-time has additional dimensions which are spontaneously compactified, so that at low energies they are seen

 © 1985 American Institute of Physics

only as additional fields and their interactions. It is ordinarily assumed that the additional dimensions are spacelike; i.e., one considers a manifold M^d with a metric g_{MN} of signature $(- + + \cdots +)$. However, instead of fixing the signature of the metric beforehand, it would appear desirable to determine it dynamically, allowing the possibility, in principle, of additional timelike dimensions. The hypothesis of the existence of additional compactified timelike dimensions was discussed in Ref. 4. It was shown in Ref. 5 that there exist solutions of an effective field theory of a superstring in a space with additional timelike dimensions.

In general, the existence of additional timelike dimensions leads to difficulties related to the appearance of ghosts and tachyons in the effective four-dimensional theory. In the present letter we point out some topological criteria on compactification under which there are no ghosts, and under which it becomes possible to derive a satisfactory four-dimensional theory of massless interacting fields. It is also assumed that a cutoff is introduced in the theory at large three-dimensional momenta, so that tachyons do not arise at energies below Planckian energies.

We consider a manifold M^d with a metric g_{MN} of signature $(d-3, 3)$, which corresponds to $d-3$ timelike dimensions. We consider a reduction of the form $M^d = M^2 \times B^{d-4}$, where M^4 is a space-time with signature (1, 3), and B^{d-4} is a compact space. If the four-dimensional theory is to have no massless ghosts, we impose the following conditions on B: (1) B has no Killing vector fields. (2) If the matter fields include skew-symmetric tensors of rank r, the Betti number is $b_{2k+1} = 0$ for $k = 0, 1, \ldots; 2k+1 \leqslant r$.

Condition (1) prevents the appearance of ghost gauge fields (although massless gauge fields do not always correspond to isometries), while condition (2) means that there are no harmonic forms of odd rank, so that there are no corresponding ghost fields. We now consider some examples that satisfy these criteria.

1. A pure gravity with Einstein's equations for the metric in the space M^d of signature $(d-3, 3)$. There is a vacuum solution corresponding to the compactification $M^d = M^4 \times B^n$, $n = d-4$, where M^4 is the Minkowski space, and B^n is a Ricci-planar compact manifold which has no Killing fields. With $n = 4$ for example, we can take $B^4 = K3$, where $K3$ is the four-dimensional Kummer surface; for $n = 6$ we could take B^6 to be the Kalabi-Yau space.[3] The effective low-energy four-dimensional theory of zeroth-order modes describes the interaction of the gravitational field with massless scalar fields.

2. The equations of a gravitational field with a Λ term, $R_{MN} = \Lambda g_{MN}$, $\Lambda > 0$, in a $(4+n)$-dimensional space-time with signature $(n+1, 3)$ allow the compactification $dS^2 \times \Pi^n$, where dS^4 is a four dimensional de Sitter space, and Π^n is an n-dimensional compact manifold of constant negative curvature. Examples of such manifolds are given in Ref. 7; for $n = 2$, for example, Π^2 is a sphere with two handles. This compactification satisfies criterion (1), since the compact hyperbolic spaces do not have vector Killing fields according to the Bochner theorem.

3. The interaction of gravity with a Yang-Mills field in M^7 with signature (4, 3). There is a compactification $a dS^4 \times \Pi^3$ through an immersion of the Yang-Mills field in a spin connection, where Π^3 is a three-dimensional compact manifold of constant

negative curvature with $b_1 = 0$, e.g., a hyperbolic space of a dodecahedron (cf. the compactification on a space of positive curvature[9]).

4. For an 11-dimensional supergravity in the signature (8, 3) we use the Freund-Rubin ansatz.[8] We find the compactification $M^{11} = a\mathrm{dS} \times B^7$ without ghosts, where B^7 is a compact manifold of constant negative curvature with $b_1 = b_3 = 0$.

5. We can now show that the introduction of additional timelike dimensions makes it possible to find a nontrivial vacuum solution in the boson sector of a $d = 10$, $N = 1$ supergravity which is interacting with a Yang-Mills field with gauge group G that contains O(3). This vacuum solution solves the problem of the cosmological constant. We have in mind a theory which becomes a supersymmetry theory after a Euclidean rotation of the additional spacelike directions. The boson part of the action is given by the Green-Schwarz Lagrangian[2]

$$\int d^{10}x e\left[\frac{1}{2k^2} R - \frac{1}{k^2}(\nabla\varphi)^2 - \frac{e^{-\varphi}}{4g^2} F^2 - \frac{3k^2}{2g^4} e^{-2\varphi} H^2\right], \tag{1}$$

where $H = dB - \omega_{3Y} + \omega_{3L}$, $\omega_{3Y}(\omega_{3L})$ is the Chern-Simon form for gauge group G (the Lorentz group), and the notation is otherwise that of Ref. 2. In the signatures $(-+++------)$ and $(-+++---+++)$, the boson equations are solved through the following substitutions, respectively: $g_{MN} = (g_{\mu\nu}(x), g_{mn}(y), g_{ab}(z))$, $H_{mnk} = \sqrt{-g(y)} h \epsilon_{mnk}$; $m, n, k, = 4,5,6$; $F^{ij}_{ab} = R^{ij}_{ab}$; $a, b = 7,8,9$; $i,j = 1,2,3$; the other components of H and F are zero; and $\varphi(x) = \varphi_0$. This substitution leads to the following equations on the metric: $R_{\mu\nu} = 0$, $R_{mn} = -\frac{3}{4}(g^2/k^2) e^{\varphi_0} g_{mn}$, $R_{ab} = (g^2/k^2) e^{\varphi_0} g_{ab}$. As the solutions of these equations in the two signatures under consideration here, (7, 3) and (4, 6), we can take $M^{10} = M^4 \times Q^3 \times \Pi^3$ and $M^{10} = M^4 \times Q^3 \times S^3$, respectively, where M^4 is the Minkowski space, and $Q^3 = S^3/\Gamma$, where Γ is a discrete subgroup of SO(4) such that Q^3 has no vector Killing fields, S^3 is a three-dimensional sphere, and Π^3 is a three-dimensional compact hyperbolic manifold.

We note that a compactification onto a planar space is possible by virtue of the original sypersymmetry of the theory. A perturbation of Lagrangian (1), e.g., the replacement of the term $e^{-\varphi} F^2$ by $e^{-q\varphi} F^2$ (q is a constant), will lead to a nonplanar four-dimensional space.

We wish to thank V. N. Berezin and V. P. Frolov for discussions.

[1]A. Salam, "Kaluza-Klein proposal and electro-nuclear gravity" (Trieste Preprint, IC/84/170).
[2]M. B. Green and J. H. Schwarz, Phys. Lett. B **149**, 117 (1984).
[3]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, "Vacuum Configurations for Superstrings," Princeton preprint, 1984.
[4]A. D. Sakharov, Zh. Eksp. Teor. Fiz. **87**, 375 (1984) [Sov. Phys. JETP **60**, 214 (1984)].
[5]I. Ya. Aref'eva and I. V. Volovich, Teor. Mat. Fiz. **64**, 300 (1985).
[6]M. J. Duff, B. E. W. Nilsson, C. N. Pope, and N. P. Warner, Phys. Lett. B **149**, 90 (1984).
[7]W. P. Thurston, Bull. Am. Math. Soc. **6**, 357 (1982).
[8]P. G. O. Freund and M. Rubin, Phys. Lett. **97B**, 233 (1980).
[9]D. V. Volkov and V. I. Tkach, Teor. Mat. Fiz. **51**, 171 (1982).

Translated by Dave Parsons