Orientational phase transition near a vortex in ^{3}He-B

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It is shown theoretically that a phase transition occurs in ^{3}He-B and gives rise to an inhomogeneous texture of the field of the directrix **n** near a singular vortex. The possibility that it is this transition which was recently observed in NMR experiments in rotating ^{3}He-B is discussed.

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A phase transition has been observed[1] in an NMR study of rotating ^{3}He-B at $T/T_c = 0.6$. The transition temperature was independent of the rotation velocity, showing that the transition is associated with a single vortex. Volovik and Mineev[2] have suggested that this transition results from a change in the structure of the vortex core, possibly involving a transition from the B phase to the A phase in it. In this letter we wish to call attention to another type of phase transition which involves a single vortex. This transition involves not a change in the structure of a core but a change in the texture of the field of the directrix **n** around the vortex.

Our analysis is carried out in the Ginzburg-Landau region. We assume that a magnetic field is directed along the axis of the vortex (the z axis). Far from the vortex, the directrix **n** must be directed along the magnetic field, but as we move closer to the vortex the directrix senses the orienting effect of the superfluid flows,[3–5] which tend to deflect **n** away from the z axis. We assume for simplicity that the projection of **n** onto the xy plane which results from this deflection does not rotate in this plane (we assume that it is directed along, say, the x axis), and we assume that the angle (β) between **n** and the z axis depends on only the distance (r) from the line of the vortex. Working from the known expressions[3–5] for the orientational energy and the deformation energy of the **n** field, we write the energy density for small deflection angles β as follows:

$$f = aH^2\left\{R_H^2\,(d\beta/dr)^2 - \frac{\beta^2}{2}\left(1+\frac{1}{h^2}\right)(r^*/r)^2 + \beta^2\right\}, \tag{1}$$

where

$$R_H^2 = \frac{12}{13}\frac{c}{aH^2} = R_{H_0}^2\,\frac{\tau}{h^2}, \qquad \tau = 1 - \frac{T}{T_c}, \quad h = H/H^*, \tag{2}$$

$r^* = 1.3\times10^{-3}$ cm, $H^* = 21$ Oe, and $R_{H_0} = 1.2$ cm. The energy associated with the orientation caused by the superfluid flow [the second term in (1)] increases in proportion to $1/r^2$ and becomes larger than the energy associated with the orientation by the magnetic field [the third term in (1)] at small values of r. In deriving (1) we took an average over the angle between the superfluid velocity and the projection of **n** onto the xy plane.

Varying the free energy with respect to β we find

$$-\frac{d^2\beta}{d\rho^2} - \frac{1}{\rho}\frac{d\beta}{d\rho} - \frac{\alpha^2}{\rho^2}\beta + \beta = 0, \tag{3}$$

where

$$\rho = r/R_H, \quad \alpha^2 = \frac{1}{2}(r^*/R_H)^2\left(1+\frac{1}{h^2}\right). \tag{4}$$

Equation (3) is equivalent to a two-dimensional Schrödinger equation for a particle in an attractive potential $1/r^2$ with an energy $E = -1$. If this Schrödinger equation has solutions which vanish at infinity (bound states) and correspond to a negative energy $E < -1$, then the uniform **n** field is unstable with respect to small deflections from the z axis (small values of β). We know that a particle in an attractive potential $1/r^2$ may fall to the center; i.e., there are bound states with arbitrarily high negative energies.[6] In our problem, however, this potential is cut off at the dimension of the vortex core, r_0, so that a state with $E \leqslant -1$ arises only at a certain value of α. The core effect can be taken into account by a boundary condition at $\rho = \rho_0 = r_0/R_H$ for Eq. (3):

$$\left.\frac{\beta'}{\beta}\right|_{\rho=\rho_0} = \frac{\alpha^2}{\rho_0}\,\kappa, \qquad (5)$$

Since the effective potential for β in the core must be on the order of α^2/ρ_0^2, the constant κ in (5) is on the order of unity.

A solution of Eq. (3), which falls off at infinity, is the modified Hankel function of imaginary index:

$$\beta \sim K_{i\alpha}(\rho) \underset{\rho \ll 1}{\rightarrow} -\frac{\sin(\alpha \ln \rho)}{\alpha}. \qquad (6)$$

Boundary condition (5) is satisfied at a discrete set of α values, the smallest of which determines the stability boundary of the uniform state, $\beta = 0$:

$$\alpha_0 = \frac{\pi/2}{|\ln \rho_0| + \kappa}. \qquad (7)$$

At $\alpha > \alpha_0$ the ground state corresponds to a nonuniform texture of the **n** field, whose form can be determined by working from the expression for the energy which is nonlinear in β. The first few nonlinear corrections which we determined show that unless the nonlinearities in the core are taken into account ($\kappa = \text{const}$) the phase transition at $\alpha = \alpha_0$ is a second-order transition.

Substituting into (7) the values of the constants from (2) and (4), and ignoring the numbers in comparison with the large logarithm, we find the following equation for the phase-transition line in the variables h and τ (Fig. 1):

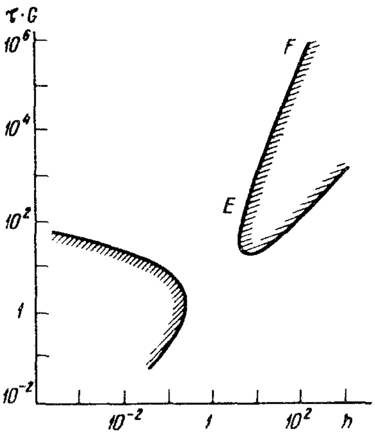

FIG. 1. Phase diagram in terms of the variables τ and h (logarithmic scale). The hatching shows the regions in which the homogeneous state, $\beta = 0$, is unstable; the point corresponds to the conditions of the experiments of Ref. 1, where a phase transition was observed.

$$\sqrt{\frac{1+h^2}{G\tau}} = \frac{\pi/2}{\ln(G\tau/h)}, \quad G = \frac{1}{2}(r^*/R_{H_0})^2. \tag{8}$$

The large constant $G \sim 10^6$ is determined by the ratio of the depairing energy to the dipole energy. In the region $G\tau \gg h \gg 1$ (branch EF in Fig. 1) we find the analytic expression $h \cong \pi\sqrt{G\tau}/\ln(G\tau)$ for the critical field h. Although this field is quite strong, it may still be weaker than the field $h \sim \sqrt{G\tau}$, at which the transition from the B phase to the A phase occurs, since $\ln(G\tau)$ is quite large.

Up to this point we have considered the transition involving only a single vortex. If there is a finite concentration of vortices, the condition that β fall off at infinity must be replaced by the condition that β' vanish at the boundary of the vortex-lattice cell, centered at the vortex, and we must seek β as a superposition of two independent solutions of Eq. (3). The phase diagram which we have derived here is valid as long as the distance (b) between vortices satisfies

$$b^2 \gg R_H r^* \left(1 + \frac{1}{h^2}\right). \tag{9}$$

At small values of b (high rotation velocities), which satisfy the opposite inequality, the condition for the stability of the $\beta = 0$ state becomes the condition derived by Gongadze *et al.*,[4] who assumed that β does not vary within the vortex-lattice cell.

Let us compare the phase transition of this paper with that which was observed experimentally. According to our estimates, condition (9) was satisfied experimentally, so that the parameters of the transition should be only slightly dependent on the rotation velocity according to our theory; this conclusion agrees with experiment. At the transition point, however (the point in Fig. 1 corresponds to the experimental condition), the experimental magnetic field is 11 times weaker than the theoretical value (branch EF in Fig. 1). Furthermore, the experimental phase transition was a first-order rather than second-order transition. Nevertheless, we do not believe that these discrepancies rule out the possibility that the orientational transition discussed here is somehow related to the transition observed experimentally. In the first place, the theory was derived for the Ginzburg-Landau region, while the experimental conditions did not correspond to this region. Second, if we discard our simplifying assumptions (ignoring rotations of $\mathbf{n}$ in the xy plane and ignoring the dependence of β on the azimuthal angle) we should find a larger instability region for the homogeneous state and, possibly, a change in the order of the phase transition. Finally, an orientational phase transition and a phase transition inside a core[2] are not necessarily mutually exclusive, since the two can occur simultaneously. A more serious discussion of this possibility will require a quantitative theory for the phase transition inside the core, not presently available. We thus cannot say with any certainty whether the orientational phase transition corresponds to the transition observed experimentally or whether it should be observed as yet another phase transition.

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