Plasma pressure surges due to the growth of ballooning modes

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A system of equations is proposed to explain the bursts of MHD waves observed in experiments on producing a high-pressure plasma in a tokamak through neutral injection.

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Increasing the plasma pressure is a central problem in research on magnetic-confinement controlled fusion. Recent experiments on the production of high-pressure plasmas in tokamaks through neutral injection[1,2] have reached $\beta \sim 2$–3%, where $\beta = 8\pi p/B^2$ is the ratio of the plasma pressure to the magnetic pressure; a working reactor would require $\beta \sim 5$–6%. Experiments have now reached a pressure range where a disruption of the magnetic configuration by a ballooning-mode instability has been predicted theoretically.[3] Even the early experiments carried out in the T-11 tokamak[1] revealed an elevated MHD activity, but this activity was seen particularly clearly in the PDX devices,[2] where x-ray detectors, devices detecting fast charge-exchange neutrals, and magnetic probes all revealed a series of bursts of oscillations (Fig. 1). The typical duration of a burst was $\tau \lesssim 0.5$ ms, and the interval between bursts was $T \sim 4$ ms. These bursts are apparently due to the growth of low-order MHD ballooning modes.[2,4] One is struck by their very nonlinear nature, which deserves a more detailed theoretical analysis.

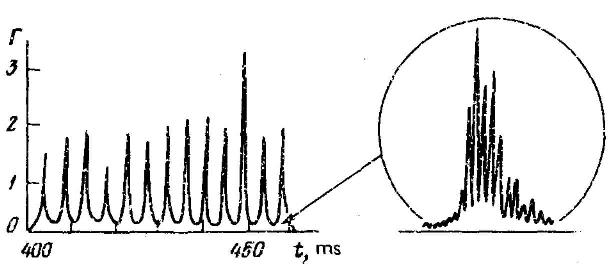

FIG. 1. Flux of fast neutrals found in the PDX device.[2]

In this letter we propose a system of equations to explain the observed explosive nature of the onset of this instability. There are three equations in this system.

The first, describing the increase in the wave energy W due to the onset of the ballooning-mode instability, is

$$\frac{dW}{dt} = 2\gamma\beta^* \left(\frac{\beta}{\beta^*} - 1\right) W - \frac{1}{\tau_W} W. \tag{1}$$

The first term on the right side of (1) is responsible for the growth of waves with a growth rate γ when the plasma pressure β exceeds a certain critical β^*. The plasma is assumed to consist of a cold (bulk) component β_c and a hot component generated by the neutral beam, β_h; the numerical factor σ is a measure of the importance of the hot component in the total pressure: $\beta = \beta_c + \sigma\beta_h$ $(0 \leqslant \sigma \leqslant 1)$. Both γ and β^* can be determined from the theory of ballooning modes.[3,5] For high-order wave modes, these values are well known. At present, there is some uncertainty regarding the $m = 1$ mode. At $\beta < \beta^*$ the growth rate is $\gamma = 0$, and the second term on the right side of (1) describes wave damping over a scale time τ_W. This is the familiar damping of Alfvén waves in an inhomogeneous plasma due to the scattering of a wave packet.[6] For an estimate we can adopt $\tau_W \sim 10\ a/c_\theta$, where $c_\theta = \beta_\theta/\sqrt{4\pi\rho}$—is the Alfvén velocity calculated from the field of the current. The operator τ_W is actually nonlinear, since at $\gamma > 0$ we have $\tau_W = \infty$, and the damping $\tau_W \sim 10\ a/c_\theta$ arises only at $\gamma = 0$, but in practice this nonlinearity is of minor importance.

The second equation is written for the pressure of the bulk component of the plasma:

$$\frac{d\beta_c}{dt} = \frac{1}{\tau_t}\beta_h - \frac{1}{\tau_E}\beta_c - \frac{1}{\tau_c} W \beta_c. \tag{2}$$

The first term on the right describes the heating of the cold plasma due to the thermalization of the hot component β_h. At moderate energies of the beam neutrals, the scale time of this thermalization is[2] $\tau_t \sim 0.01$ s, and it generally depends on the temperature of the cold component.[7] The second term in (2) describes the ordinary energy loss in the bulk component (τ_E is the energy lifetime). In the experiments of Ref. 2 the value is $\tau_E \sim 20$ ms. The last term on the right side of (2) reflects the surge in the energy of the bulk component which results from the instability.

The third equation is written for the hot component, which is produced by the injection source of power P (all quantities here are normalized by dividing by the energy of the magnetic field, so that W and β are dimensionless, while P has the dimensionality $1/s$):

$$\frac{d\beta_h}{dt} = P - \frac{1}{\tau_t}\beta_h - \frac{1}{\tau_h} W\beta_h . \tag{3}$$

The second term on the right of (3) reflects the cooling of the hot component due to the thermalization of the beam, and the last term describes the experimentally observed burst of fast particles, which is assumed to increased with increasing wave energy.

The parameters P, τ_E, and τ_t in Eqs. (1)–(3) are well known. The parameters β^* and γ are known less accurately. The uncertainty is greatest regarding τ_c and τ_h, which describe the increase in the plasma energy. Several models can be proposed for this process, and τ_c and τ_h can be calculated from these models, but will not take this approach here; we will instead leave them as adjustable parameters. Analysis of the equations (discussed below) shows that there is a rather narrow interval of the parameters τ_c and τ_h over which the results of the integration of system (1)–(3) describe the experimental situation. All that we say at the outset is that $\tau_h < \tau_c$.

It is natural to begin the study of system (1)–(3) with the linear stage. It turns out that there exists a critical injection power above which an instability occurs. It is convenient to introduce an equilibrium wave energy density[1)]

$$W_0 = \frac{\tau_h}{\tau_t}\left[\frac{\tau_E P}{\beta^* + 1/2\gamma\tau_W} - 1\right] \tag{4}$$

and a critical wave energy $W_{cr} = \tau_h/2\gamma\tau_t^2$. At $W_0 < W_{cr}$, all three roots of the characteristic equation of the linearized system give us damped solutions; at $W_0 = W_{cr}$, two neutral oscillations appear with a frequency $\omega_{1,2} = \pm(1/\tau_E\tau_t)^{1/2}$; the third root, $\omega_3 = -i/\tau_t$, again gives us a damped solution. If[2)] $W_0 > W_{cr}$, the real roots $\omega_{1,2}$ are displaced into the complex region, and they acquire a positive imaginary increment corresponding to wave growth. If the damped root ω_3 is far greater in modulus than the growing roots, we can expand Eqs. (1)–(3) in powers of the reciprocal of this ratio. As a result, we replace these three equations by two others describing a self-excited oscillator. In this approximation the right sides of the resulting equations do not depend on the time; i.e., the system is autonomous and describes comparatively slow, periodic, nonlinear oscillations around the equilibrium energy W_0 given in (4). The situation changes if the parameters of the system are such that three roots are comparable in magnitude (in modulus). In this case we are forced to use all three equations. Figure 2a shows the time dependence of the oscillation energy over an interval of $5\tau_E$, found from the numerical integration of Eqs. (1)–(3). At the parameter values adopted, the period and length of the peaks agree satisfactorily with experiment. As further arguments for this model we might cite the approximate agreement of the critical injection power[2)] $P_{theo} \sim 2.5$ MW (at which the instability occurs) and the experimental

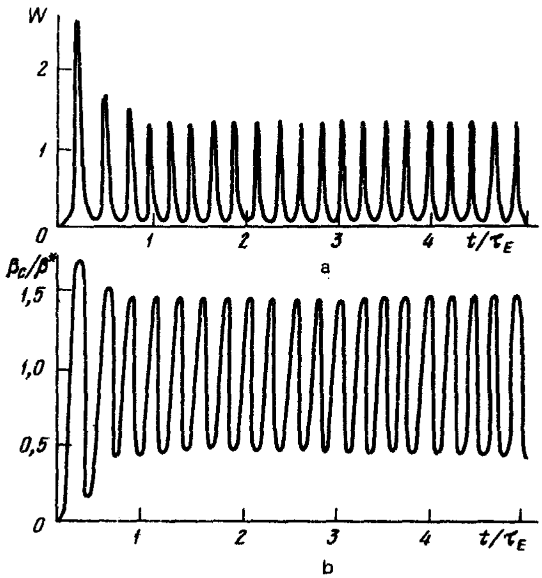

FIG. 2. Results of a numerical solution of system (1)–(3). a—Time evolution of the wave energy W (in arbitrary units) for the parameter values $\gamma\tau_E\beta^* = 400$; $P\tau_E/\beta^* = 4$; $\sigma = 0$; $\tau_W/\tau_E = 0.025$; $\tau_i/\tau_E = 0.5$; $\tau_c/\tau_E = 0.25$; $\tau_h/\tau_E = 0.1$; b—fluctuations in the pressure of the cold plasma component, β_c/β^* (in arbitrary units), for the same parameter values as in Fig. 2a.

value $P_{\rm expt} \sim 3$ MW and the fluctuations in the pressure of the cold component, $\beta_c \sim 40$–60% (Fig. 2b).

Experimentally, we find different results when the neutrals are injected parallel and antiparallel to the current. In the case of antiparallel injection, these oscillations have not yet been observed. A possible explanation for this difference is that in the latter case the orbits of the trapped beam particles are far wider than in the former case (50% of the beam particles strike the wall according to an estimate in Ref. 2). The pressure profiles may be flatter, and conditions may hinder the growth of ballooning modes.

It thus appears that this simple new model reflects the basic qualitative features of the observed nonlinear bursts of oscillations and hot particles in the PDX device.

[1)]For simplicity, we are assuming $\tau_c \to \infty$.
[2)]In practice, the condition $W_0 > W_{\rm cr}$ reduces to $\tau_E P > \beta^*$.

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