Proton decay in grand unified models with horizontal symmetry

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If a horizontal symmetry exists between quark-lepton generations, the decay of the proton in any grand unified model should go through the Higgs channel (through scalar colored triplets) at rates of the same order of magnitude as in the vector channel (through X and Y bosons).

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In any standard grand unified model[1]—SU(5), SO(10), E(6)—which independently describes each of the quark-lepton generations,

$$(u, d; e, \nu_e), \quad (c s; \mu \nu_\mu), \quad (t\,?, b; \tau, \nu_\tau), \tag{1}$$

the decay of the proton through the exchange of Higgs scalars is suppressed in comparison with the ordinary decay through gauge fields (X and Y bosons) which have acquired masses as a result of the spontaneous breaking of the original symmetry of the grand unified model, Γ:

$$\Gamma \to SU(3)_c \otimes SU(2) \otimes U(1). \tag{2}$$

To support this assertion, we note that the ratio of effective constants of the four-fermion interactions leading to the decay of the proton is

$$G^S/G_V = \frac{f_q^2}{\bar{g}^2}\frac{M^2}{M_S^2} \lesssim 10^{-5}, \quad f_q^2 = 2^{3/2} G_F m_q^2, \tag{3}$$

where $\bar{g}$ is the gauge constant of the grand unified model ($\bar{g}^2/4\pi \equiv \bar{\alpha} \simeq 0.022$), M is the mass of the X (Y) bosons $M \simeq 10^{15}$ GeV), f_q is the Yukawa constant coupling the quark $q(q = u,d)$, of mass m_q ($m_{u,d} \lesssim 10$ MeV), with a scalar color triplet[1)] of mass $M_S(\bar{\alpha}M \lesssim M_S \lesssim M)$, and G_F is the Fermi constant.

A completely different situation arises if the quark-lepton generations in (1) are linked by a horizontal symmetry H, which transforms these generations into each other.[2] In a grand unified model with a $\Gamma \otimes H$ symmetry all the Yukawa couplings are H-symmetric, so that the constants f_q for all generations of quarks and leptons in (1) are equal: $f_u = f_c = f_t \equiv f_1, f_d = f_s = f_b \equiv f_2$. The reason why masses of the different generations are different is not that the Yukawa constants differ from each other (by many orders of magnitude), as in the standard models,[1] but that there is a hierarchy of vacuum expectation values of scalars which spontaneously break the horizontal symmetry H (Ref. 3). In this case, the Yukawa constant f^2 (if we assume $f_1 \approx f_2 \equiv f$) is proportional, in contrast with f_q^2(3), to the total sum of the squares of the masses of all the quarks [including heavy c, b, and t quarks; see (15)] and is therefore large. The proton-decay channels induced by the scalars are enhanced and may even become dominant.

To illustrate these general arguments we consider the particular case of a local $SU(5)\otimes SU(3)_H$ symmetry, which we have studied previously.[2,3] In this model, the three generations of quarks and leptons in (1) fill left-hand spiral multiplets

$$\Psi^{i\alpha}(\bar{5},\bar{3}),\quad \Psi^{\alpha}_{[ij]}(10,\bar{3}), \tag{4}$$

where $i(1,\ldots,5)$ and $\alpha(1,2,3)$ are the indices of the SU(5) and $SU(3)_H$ groups, respectively [the dimensionalities of the representations are given in parentheses in (4) and everywhere below]. The masses of the up quarks $U(u,c,t)$, the down quarks $D(d,s,b)$, and the leptons (e,μ,τ) induce vacuum expection values for the scalar fields $\omega_{i\{\alpha\beta\}}(5,6)$ and $\rho^i_{\{\alpha\beta\}}(\bar{5},6)$, respectively, through the Yukawa couplings (f_1 and f_2 are constants, and c is the charge conjugation matrix)[2]:

$$f_1\,\psi^{\alpha}_{[ij]}\,C\,\psi^{\beta}_{[kl]}\,\omega_{m\{\alpha\beta\}}\,\epsilon^{ijklm},\quad f_2\,\psi^{i\alpha}\,C\,\psi^{\beta}_{[ij]}\,\rho^{j}_{\{\alpha\beta\}}. \tag{5}$$

The scalar sector of the model includes, in addition to the ω and ρ fields, the ordinary scalars[1] SU(5), $\Phi^i_j(24,1)$, and $\phi_i(5,1)$ with vacuum expectation values of the type

$$<\Phi^i_j> = V\,\mathrm{diag}\,(1,1,1,-3/2,-3/2)^i_j,\quad M=\frac{5}{2\sqrt{2}}\bar{g}V, \tag{6}$$

which cause a "breakdown" of SU(5) symmetry (2), and scalars which break the $SU(3)_H$ symmetry: the sextet $\chi_{\{\alpha\beta\}}(1,6)$ and the two triplets $\xi_\alpha(1,3)$ and $\eta_\alpha(1,3)$,

$$<\chi_{\{\alpha\beta\}}> = \mathrm{diag}\,(r_1,r_2,r_3)_{\alpha\beta},\quad <\xi_\alpha>=p\,\delta_{\alpha_1}\quad <\eta_\alpha>=q\delta_{\alpha_3} \tag{7}$$

with the horizontal hierarchy of vacuum expectation values

$$r_3\sim ap\sim a^2q\sim a^2r_2\sim a^6r_1,\ a\sim(m_c/m_u)^{1/4},\ r_3\equiv V_H\sim V, \tag{8}$$

which arise in a natural way from the Higgs potential of the fields[3] χ, ξ, and η.

The spontaneous breaking of the $SU(2)\otimes U(1)$ symmetry which remains follows from the general potential of the ϕ, ω, and ρ fields,[2] which necessarily contains all the couplings allowed by the $SU(5)\otimes SU(3)_H$ symmetry with fields which develop large vacuum expectation values (Φ, χ, ξ, and η):

$$P=P_\phi+P_\omega+P_\rho+P_{\phi\omega}+P_{\phi\rho}+P_{\omega\rho}. \tag{9}$$

After substituting the large vacuum expectation values of these fields (6,7) into the polynomial P, we find the mass matrix[3] of the fields ϕ, ω, and ρ, which contain diagonal and off-diagonal elements of order

$$M^2_\phi\sim M^2_\omega\sim M^2_\rho\approx\lambda V^2,\ M^2_{\phi\omega}\sim M^2_{\phi\rho}\approx\lambda V r_\alpha,\ M^2_{\omega\rho}\approx\lambda r^2_\alpha \tag{10}$$

($\alpha=1,2,3$; $r_3\sim V\gg r_2\gg r_1$), where λ is the natural order of the interaction constant of the scalars in (9), satisfying $\bar{\alpha}^2\lesssim\lambda\lesssim 1$. In accordance with the gauge hierarchy hypothesis,[1] we require that the mass matrix $M^2_{(2)}$ of the doublet (in the weak isospin) components of the fields ϕ, ω, and ρ have a single small negative eigenvalue, $-\mu^2$, and we require that the corresponding state $H^{(0)}$ condense with the vacuum expectation value

$$<H^{(0)}>=v,\quad v\equiv\sqrt{2\mu^2/h}=2^{3/4}G_F^{-1/2} \tag{11}$$

(h is the self-effect constant of the $H^{(0)}$ doublet). All other eigenvalues of the matrix $M^2_{(2)}$ (corresponding to $H^{(n)}$, $n=1,\ldots,12$) are large[4] $(\sim\lambda V^2)$.

As for the color triplets of the ϕ, ω, and ρ multiplets, we note that their their mass matrix $M^2_{(3)}$ diagonalizes without any special conditions, so that all its eigenvalues are large ($\sim\lambda V^2$). These triplets are responsible for the decay of the proton through the general (with doublets) Yukawa couplings (5) with the constants f_1 and f_2, to whose calculation we now turn. Expanding in terms of the states $H^{(0)}$ and $H^{(n)}$ the doublets in the scalars ϕ, ω, and ρ, for their vacuum expectations values

$$<\omega_{i\{\alpha\beta\}}> \equiv \delta_{i5}\,(v_u, v_c, v_t)^{\rm diag}_{\alpha\beta}, \quad <\rho^i_{\{\alpha\beta\}}> \equiv \delta^i_5\,(v_d, v_s, v_b)^{\rm diag}_{\alpha\beta} \tag{12}$$

[regulated by "horizontal hierarchy" (8)[3]] and $\langle\phi_i\rangle \equiv \delta_{i5} v_0$, we find (the normalization condition for the vacuum expectation values)

$$v^2 = v_0^2 + \sum_q v_q^2, \quad q = (U, D), \quad U = u, c, t; \quad D = d, s, b. \tag{13}$$

For the constants $f_1 = m_U/v_U$ and $f_2 = m_D/v_D$ of couplings (5) we then find the sum rule:

$$2^{3/2} G_F \left[\frac{1}{f_1^2} \sum_U m_U^2 + \frac{1}{f_2^2} \sum_D m_D^2 \right] \beta^{-1} = 1, \quad \beta = 1 - \frac{v_0^2}{v^2}, \tag{14}$$

which takes the following simpler form for the SO(10) for E(6) model with a single scalar multiplet[1] ($f_1 = f_2 \equiv f$)[1]:

$$f^2 = 2^{3/2} G_F \left(\sum_q m_q^2 \right) \beta^{-1}. \tag{15}$$

Retaining in sum (14) only the heavy quarks c ($m_c \cong 1.4$ GeV), b ($m_b \cong 4.5$ GeV), and t ($m_t \cong 18.5$ GeV), we find the following results for the constant f and the ratio G^S/G^V_V ($\beta \lesssim 1$):

$$f \gtrsim 0.14, \quad \bar{g} \cong 0.53, \quad G^S/G^V = \frac{f^2}{\bar{g}^2} \frac{M^2}{M_S^2} \gtrsim O(1). \tag{16}$$

The latter estimate is guaranteed to hold if the mass of the scalar is smaller than the mass of the $X(Y)$ boson, M, by a factor of at least four or five.

We have some concluding comments.

1) Estimate (16) and the experimental limit on the proton lifetime[1] requires $M_S \sim M$. This condition means that all the parameters in the potential of the scalar fields $P(9)$ and thus the self-effect constant of the $H^{(0)}$ doublet must be $\sim O(1)$, so that the Higgs boson of the Glashow-Weinberg-Salam theory must have a mass $m = O(100)$ GeV.

2) The experimental absence of a universal polarization of the charged lepton—which the exchange of X and Y bosons should cause[4]—would be evidence of an important role of the scalar interaction in the proton decay and thus indirect evidence for the existence of color scalar triplets with a mass $M_S \sim M$ and a charge of 1/3, as for the Y boson. For this reason, these triplets should increase the relative importance of proton decay channels involving the production of neutrinos.

3)We have found that in any grand unified model with horizontal symmetry the effects of the Higgs channel in the proton decay are quite important. Furthermore, all our conclusions remain in force if the horizontal symmetry $SU(3)_H$ discussed here is global rather than local.

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[1)]This $SU(3)_c$ triplet forms along with the doublet $SU(2)\otimes U(1)$ a single scalar Γ multiplet in a grand unified model [for example, a 5-plet in the SU(5) model].

[2)]Two other scalar multiplets with a triplet $SU(3)_H$ content, $(\bar{5},\bar{3})$ and $(\overline{45},\bar{3})$, are required for a realistic mass matrix of the down quarks and leptons,[3] but, for simplicity, and to save space, we will not consider them here.

[3)] We mean a (13×13) matrix of the masses of scalar 5-plets: $\phi_i(5,1)$ $\omega_{i\{\alpha\beta\}}(5,6)$ and $\rho^i_{\{\alpha\beta\}}(\bar{5},6)$. The breaking of the SU(5) symmetry in (6) lifts the mass degeneracy in the 5-plets between $SU(2)\otimes U(1)$ doublets and $SU(3)_c$ triplets and gives rise to two independent (13×13) matrices: $M^2_{(2)}$ for doublets and $M^2_{(3)}$ for triplets.

[4)]In this manner we get the Glashow-Weinberg-Salam theory[1] with a single light Higgs scalar (from the $H^{(0)}$ doublet), and the danger posed by the neutral scalar currents with a change in the fermion flavor is completely eliminated.[1]

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