Heating of a dense plasma by a powerful beam of relativistic electrons

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We indicate the conditions under which dissipation of the energy of Langmuir oscillations excited in a plasma by a relativistic electron beam leads to an effective heating of the bulk of the plasma electrons. These conditions can be realized in experiments on beam heating of a dense plasma.

When a relativistic electron beam is collectively decelerated in a plasma, its energy is transferred to Langmuir oscillations with large phase velocities (see[1–3]). Owing to the cumulation of these oscillations, a so-called modulation instability can occur in the plasma,[4] and its development causes the entire energy of the Langmuir oscillations to be absorbed by a small group of fast electrons.[5,6] Although it is not excluded that these electrons have time to transfer their energy to the principal plasma mass before they leave the installation (the concrete situation depends on the details of the containment system), it would be naturally preferable to heat all the plasma electrons from the very outset. We show in this communication that if the plasma density is high enough, then there exists a wide range of experimental conditions in which there is no modulation instability, and therefore most of the electrons become heated.

The oscillations directly excited by the beam will be called resonant. The wave vectors are of the order of $k \sim \omega_{pe}/c$. Owing to induced scattering by the ions, these oscillations become redistributed over the spectrum, shift to the region of small wave vectors, and cease to interact with the beam. This circumstance limits the energy density U_R of the resonant oscillations to the level

$$U_R \sim 10 nT \frac{\gamma}{\omega_{pe}} \frac{M}{m} \frac{T}{mc^2}, \tag{1}$$

where γ is the two-stream instability increment, T is the plasma temperature ($T_e = T_i = T$), and M and m are respectively the ion and electron masses. In a unit time, the beam loses an energy equal to γU_R, which is practically entirely transferred to the long-wave part of the spectrum. We shall henceforth be interested in the conditions under which the dissipation of the long-wave oscillations is due to Coulomb collisions. Then the energy density U_0 of the long-wave oscillations is connected with U_R by the conservation law

$$\nu U_0 = \gamma U_R, \tag{2}$$

where ν is the frequency of the collisions between the plasma electrons and the ions. If no modulation instability is produced in the system, then the energy lost by the beam is suddenly transferred to the bulk of the plasma electrons. We present the corresponding estimates.

In a plasma without a magnetic field, the stability criterion takes the form (see[4]):

$$\frac{V_0}{nT} < k_0^2 r_D^2. \tag{3}$$

Here k_0 denotes the characteristic value of the wave vector of the long-wave oscillations, and r_D is the Debye radius. Since the spectrum in a plasma without a magnetic field has a tendency to "contract" to values $k_0 \ll r_D^{-1}\sqrt{m/M}$, the condition (3) is very stringent. In fact, the only possibility is to carry out the heating under weakly supercritical conditions, when the increment of the two-stream instability is only 1.5—2 times larger than the frequency of the Coulomb collisions, and the oscillations attenuate before they reach the region $k \lesssim r_D^{-1}\sqrt{m/M}$ as they become redistributed over the spectrum.

At $\gamma \sim \nu$, the length within which the beam is stopped is, in accordance with[1-3],

$$l \sim \frac{1}{20}\ \frac{c}{\nu}\ \frac{m}{M}\left(\frac{E}{T}\right)^2 \tag{4}$$

(E is the energy of the beam electrons).

In the case of a sufficiently dense plasma, this estimate yields perfectly acceptable values of l (thus, at $n \sim n^{18}$ cm^{-3}, $T \sim 10^4$ eV, and $E \sim 10^6$ eV we have $l \sim 2$ meters for a deuterium plasma).

The situation is more favorable in the presence of even a relatively weak magnetic field in the plasma. The reason is that the field changes the dispersion of the Langmuir oscillations, which now takes the form

$$\omega(\mathbf{k}) = \omega_{pe}\left[1 + \frac{3}{2}k^2 r_D^2 + \frac{1}{2}\ \frac{\omega_{He}^2}{\omega_{pe}^2}\ \frac{k_\perp^2}{k^2}\left(1 - \frac{\omega_{pe}^2}{k^2 c^2}\right)\right] \tag{5}$$

(formula (5) is valid so long as the "magnetic" increment to the frequency is small in comparison with the cyclotron frequency ω_{He} of the electrons). If $8\pi nT/H_2 \gtrsim 1$, then the field influences only the structure of the long-wave part of the spectra ($k \ll \omega_{pe}/c$), and the spectrum of the resonant oscillations remains practically unchanged (in particular, their energy is given by formula (1) as before). The change of the dispersion relation for long-wave oscillations leads to two essential effects. First, owing to the increase of the dispersion increment to the frequency, an increase takes place in the time $\tau(k_0)$ of the spectral transfer of the oscillations from $k \sim \omega_{pe}/c$ to $k_0 \ll \omega_{pe}/c$, and by the same token the role of the collision dissipation increases. Second, the criterion for the absence of modulation in stability becomes less stringent. This criterion is essentially the requirement that the nonlinear increment to the frequency be small in comparison with the dispersion increment

$$\frac{U_0}{nT} < \frac{\omega_{He}^2}{\omega_{pe}^2}\ \frac{\omega_{pe}^2}{k_0^2 c^2}. \tag{6}$$

The value of k_0 in this formula is determined from the condition

$$\tau(k_0)\nu \sim 1. \tag{7}$$

An estimate of τ can be obtained from the equation describing the induced scattering of the Langmuir oscillations by ions (see[7,8])

$$\frac{\partial}{\partial t}\ln W_{\mathbf{k}} = -\frac{\pi\omega_{pe}}{2nM}\int\left(\frac{\mathbf{k}\mathbf{k}'}{kk'}\right)^2 W_{\mathbf{k}'}\,\delta'\left[\frac{\omega(\mathbf{k}') - \omega(\mathbf{k})}{|\mathbf{k} - \mathbf{k}'|}\right]d^3\mathbf{k}' \tag{8}$$

(W_k is the spectral energy density of the oscillations, and the prime at the δ function denotes differentiation with respect to the argument). From this we have

$$\tau^{-1}(k_0) \sim \omega_{pe}\frac{U_0}{nT}\left(\frac{\omega_{pe}}{\omega_{He}}\right)^4\left(\frac{k_0 c}{\omega_{pe}}\right)^6\frac{T}{Mc^2} \tag{9}$$

and accordingly

$$k_0 \sim \frac{\omega_{pe}}{c}\left(\frac{\omega_{He}}{\omega_{pe}}\right)^{2/3}\left(\frac{Mnc^2}{U_0}\right)^{1/6}\left(\frac{\nu}{\omega_{pe}}\right)^{1/6} \tag{10}$$

Combining relations (1), (2), (6), and (10) we obtain finally the stability criterion

$$\left(\frac{\gamma}{\nu}\right)^{4/3} < \frac{m}{M}\ \frac{\omega_{pe}}{\nu}\left(\frac{H^2}{8\pi nT}\right)^{1/3}. \tag{11}$$

It is easy to verify that this inequality admits of a rather large excess of γ over ν. Thus, for example, at $n \sim 10^{18}$ cm^{-3} $T \sim 10^{-4}$ eV and $H^2/8\pi nT \sim 10^{-2}$ it yields $\gamma/\nu \lesssim 20$ for a deuterium plasma.

We have thus shown that in the case of a sufficiently dense plasma it is possible to carry out the heating under conditions such that the energy lost by the beam is known to be transferred to the bulk of the electrons.

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