Quark structure of resonances as a consequence of spontaneous vacuum transitions in dual models

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(Submitted February 22, 1975)
ZhETF Pis. Red. **21**, No. 7, 454–457 (April 5, 1975)

It is shown that spontaneous vacuum transitions in the dual Veneziano model lead to the appearance of internal quantum numbers and, depending on the intercept of the Regge trajectories a_0, to one group of internal symmetries or another.

When internal quantum numbers of resonances are introduced in dual models, it is customary to use the so-called Chan-Paton procedure,[1] which ensures both invariance to the considered symmetry group and the absence of exotic resonant states.

We wish to point out here that the dynamics of dual interactions, without any preliminary introduction of internal symmetries, contains a mechanism connected with the presence of spontaneous vacuum transitions, which leads to the appearance of internal quantum numbers and, in the particular case of the intercept $a_0 = 1$, to a structure of interactions of the Chan-Paton type.

We consider a dual amplitude in the presence of induced vacuum transitions between particles i and $i+1$,[2–4]

$$\bar{B}_n(p_1, p_2, \ldots p_n) = \sum_{N_i=0}^{\infty} \beta^{N_i} B_{n+N_i}(p_1, \ldots, p_i, \overbrace{0, \ldots, 0}^{N_i}, p_{i+1}, \ldots, p_n), \tag{1}$$

which can be transformed into

$$\bar{B}_n(p_1, \ldots, p_n) = \frac{1}{\Omega} \int \frac{\prod_j dx_j}{\prod_j (x_{j+2} - x_j)} \prod_{\substack{j,k=1 \\ j<k}}^{n} (U_{j,k})^{-a_{j,k}-1} \times R(\beta, a(p_i^2), a(p_{i+1}^2), U_{lm}), \tag{2}$$

where

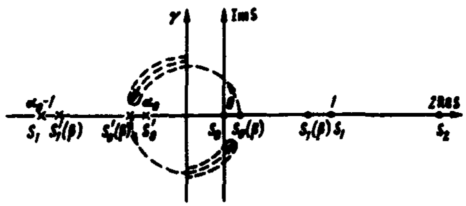
$$R(\beta, a(p_i^2), a(p_{i+1}^2), U_{lm}) = -\frac{1}{2\pi i} \frac{1}{\Gamma(-a(p_i^2))\Gamma(-a(p_{i+1}^2))} \times \int_\gamma ds_i K(a_0, s_i) \sum_{l_i=0}^{\infty} \frac{\Gamma(l_i + s_i - a(p_i^2))\Gamma(l_i + s_i - a(p_{i+1}^2))}{l_i! \Gamma(l_i + 1 - a_0 + 2s_i)} \times \prod_{j \neq i, i-2} (U_{i+1,j})^{l_i + s_i} \tag{3}$$

and

$$K(a_0, s_i) = \frac{(a_0 - 2s_i)\Gamma(-s_i)\Gamma(-a_0 + s_i)}{1 - \beta B(-s_i, -a_0 + s_i)};$$

$$a_{0j} = a(s_{ij}) = a_0 + a' s_{ij}, \tag{4}$$

where $B(x, y)$ is the Euler B function.



The integration contour is shown in the figure.

The analytic properties of the representation (3) relative to the parameter β are determined completely by the behavior of the zeroes of the denominator in expression (4) as this parameter is continuously varied.

At $\beta = 0$, the equation

$$1 - \beta B(-s_i, -a_0 + s_i) = 0 \tag{5}$$

has an infinite number of roots located at the points $s_n = n$, $s'_n = a_0 - n$, $n = 0, 1, 2, \ldots$, corresponding to the poles of the functions $\Gamma(-a_0+s_i)$ and $\Gamma(-s_i)$.

At $\beta\neq 0$, the functions $s_n(\beta)$ and $s_n'(\beta)$, which are solutions of (5) and coincide with s_n and s_n' at $\beta=0$, correspond to different Riemann branches of the infinitely-valued function $s(\beta)$. On going around some branch point, these functions experience a certain permutation. In particular, on going around a branch point located on the zeroth sheet at the point $\beta_0=\Gamma(-a_0)/[\Gamma(-a_0/2)\times\Gamma(-a_0/2)]$, permutation of the branches $s_0(\beta)$ and $s_0'(\beta)$ takes place. This bypass corresponds to the integration-contour change shown in the figure, which leads to the additional contribution

$$\frac{\Gamma(1-a_0)}{\Gamma(-a(p_i^2))\Gamma(-a(p_{i+1}^2))}\left\{\sum_{l_i=0}^{\infty}\frac{\Gamma(l_i+a_0-a(p_i^2))\Gamma(l_i+a_0-a(p_{i+1}^2))}{\Gamma(l_i+1+a_0)\,l_i!}\times\prod_{j\neq i,i-1}(U_{i+1,j})^{l_i+a_0}\right.$$
$$\left.-\sum_{l_i=0}^{\infty}\frac{\Gamma(l_i-a(p_i^2))\Gamma(l_i-a(p_{i+1}^2))}{\Gamma(l_i+1-a_0)\,l_i!}\prod_{j\neq i,i-1}(U_{i+1,j})^{l_i}\right\} \tag{6}$$

to (3) at $\beta=0$ on the first sheet of the Riemann surface in comparison with the value of the integral on the zero sheet.

The sum of (3) and (6) can be transformed into

$$\frac{\Gamma(1-a_0)}{\Gamma(-a(p_i^2))\Gamma(-a(p_{i+1}^2))}\times\left\{\sum_{l_i=0}^{\infty}\frac{\Gamma(l_i+a_0-a(p_i^2))\Gamma(l_i+a_0-a(p_{i+1}^2))}{\Gamma(l_i+1+a_0)\,l_i!}\prod_{j\neq i,i-1}(U_{i+1,j})^{l_i+a_0}\right.$$
$$\left.-\sum_{l_i=0}^{\infty}\frac{\Gamma(l_i+1-a(p_i^2))\Gamma(l_i+1-a(p_{i+1}^2))}{\Gamma(l_i+2-a_0)(l_i+1)!}\prod_{j\neq i,i-2}(U_{i+1,j})^{l_i+1}\right\}, \tag{7}$$

from which it follows that the trajectories of the external ith and $(i+1)$-st particles, and also of all the internal resonant states connected with the variables U_{i+1}, j $(j\neq i,i+2)$ splits into two, the first of these trajectories being shifted by $-\alpha_0$, and the second by -1 in comparison with the initial trajectory. The shift of the intercept of the external particles is determined in this case by the factor $\Gamma(a_0-a(p_i^2))$, and the shift of the internal trajectories is determined by the factor U_{i+1j} in (7).

The summation of the induced vacuum transitions that occur between all the remaining external particles is carried out in analogous fashion. The factor R that results on going to the first sheet at $\beta=0$ takes the form

$$\sum_{s_1,\dots,s_n=1,a_0}\prod_i C(s_i)\sum_{l_1,l_2,\dots l_n=0}^{\infty}\times\frac{\Gamma(l_{i-1}+l_i+s_{i-1}+s_i-a(p_i^2))\Gamma(l_i+l_{i+1}+s_i+s_{i+1}-a(p_{i+2}^2))}{\Gamma(l_i+1+s_i)\Gamma(l_i+1-a_0+s_i)}\times\prod_{j<k}U_{jk}^{l_j+s_j+l_k+s_k}, \tag{8}$$

where $C(1)=-1$ and $C(a_0)=1$, $(a_0\neq 1)$, from which it follows that all the external and internal trajectories are split into four trajectories, with resultant values of the intercepts a_0-2, -1, -1, and $-a_0$. The fact that the intercept -1 corresponds to two independent trajectories is the consequence of a limiting operation, in which the separation of the resonant states is carried out prior to the taking of the limit as $\beta\to 0$, during each stage of the summation of the induced vacuum transitions between any two neighboring particles.

At $0<a_2<2$, the tachyon state is eliminated.

At $a_0>2$, on the split trajectory with intercept a_0-2, the tachyon state is preserved. This state can be eliminated by an analogous procedure via allowance for the additional spontaneous transitions into the vacuum of the particles that lie on this trajectory.

As a_0 tends to unity, all four split trajectories acquire the same intercept. Simple relations are then established between the matrix elements for the different particles on these trajectories, and these relations, following a suitable multiplicative renormalization of the wave functions of the external particles, are equivalent to factorization of new degrees of freedom in the dual amplitudes in the form of factors

$$\mathrm{Sp}\,A_1A_2\dots A_n,$$

where each of the matrices A_i takes the form

$$A=\begin{pmatrix}\frac{1}{\sqrt{2}}a_0+\frac{1}{\sqrt{2}}a_3 & a^*\\ a & \frac{1}{\sqrt{2}}a_0-\frac{1}{\sqrt{2}}a_3\end{pmatrix}.$$

Thus, at $a_0=1$, the spontaneous vacuum transitions leads to the appearance of $SU(2)$ symmetry with absence of exotic states, in accordance with the Chan-Paton procedure and the Harari-Rosner method of quark diagrams.[5]

A generalization of the foregoing analysis to include the Neveu-Schwartz dual model[6] leads to the inclusion into the scheme of spontaneous vacuum transitions of an additional quantum number of g-parity, while further generalization with allowance for a symmetry of the Shapiro-Virasoro type[7] and Franke-Manida type[8] is apparently in a position to establish apperance of a quark structure of the type $SU(3)$ and higher symmetries.

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