Plasma collapse in a magnetic field

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The theory of plasma collapse[1-7] has developed so far without allowance for the influence of the magnetic field on this phenomenon. This is permissible if the energy density W of the oscillations satisfies the condition $W/nT \gg \omega_{He}/\omega_p$, where ω_{He} and ω_p are the electron Larmor and Langmuir frequencies. No less important is the opposite limiting case

$$\frac{W}{nT} \ll \frac{\omega_{He}}{\omega_p} \ll 1, \quad (1)$$

which can be easily realized both in laboratory and astrophysical conditions.

At $\omega_{He} \ll \omega_p$, there exist in the plasma three high-frequency oscillation modes $\omega_i(k)(i=1,2,3)$, viz., $\omega_1(0) = \omega_p - \omega_{He}/2$, $\omega_2(0)=\omega_p$, and $\omega_3(0)=\omega_p+\omega_{He}/2$. With a suitable choice of the initial conditions, collapse at each of these modes is possible.

If the plasma turbulence is the result of the development of weak or moderate instabilities in the region of large wave numbers, the collapse is preceded by a stage of weakly-turbulent relaxation, during which the three modes exchange energy intensively. Regardless of the actual decisive weakly-turbulent mechanism, the est-frequency (slow extraordinary) mode, which has at $\omega_1(k) - \omega_1(0) \approx \omega_p W/nT \ll \omega_{He}$ a dispersion law

$$\omega_1(k) = \omega_p - \frac{\omega_{He}}{2} + \frac{1}{4\omega_p}(c^2 + 3v_{Te}^2)^2 k_\perp^2 + \frac{c^2}{4\omega_p} k_z^2 . \quad (2)$$

The magnetic field is directed along the z axis. By averaging in succession the hydrodynamic plasma equations first over ω_p and then over ω_{He}, we obtain the equation for the complex amplitude $\psi = \exp[-i(\omega_p - \omega_{He}/2)t] \times (E_x + iE_i)/\sqrt{2}$, which corresponds to the dispersion law (2) and takes into account the weak inhomogeneity of the plasma.

$$i\psi_t + \frac{1}{4\omega_p}(c^2 + 3v_{Te}^2)\Delta_\perp\psi + \frac{c^2}{2\omega_p}\psi_{zz} = \frac{\omega_p}{2n_0}\delta n\,\psi . \quad (3)$$

To make Eq. (3) closed, it is necessary to use the wave equations for the ions

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\Delta\right)\delta n = \frac{1}{16\pi M}\Delta|\psi|^2 , \quad (4)$$

if $n_t/n \gg \omega_{Hi}$, or the "magnetized" wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\frac{\partial^2}{\partial z^2}\right)\delta n = \frac{1}{16\pi M}\frac{\partial^2}{\partial z^2}|\psi|^2 , \quad (5)$$

which is valid in the opposite limiting case. Either system, (3) and (4) or (3), (5), describes collapse. Qualitatively, the collapse pictures are the same in both cases. At very small field amplitudes ($W/nT \ll T/Mc^2$) (cf.[7]), static collapse occurs when $\delta n = -n_0|\psi|^2/8\pi$, and Eq. (3) reduces to the three-dimensional "parabolic" self-focusing equation analyzed in[1,2,8]. As a result of the collapse, an ellipsoidal cavern (spherical in the particular case $3v_{Te}^2 \approx c^2$) is produced, with a field maximum at the center and with a characteristic dimension $L \approx (c/\omega_p)(W/nT)^{-1/2}$. If $W/nT \gg T/Mc^2$, a self-similar supersonic collapse takes place (cf.[1,2,5,6], which the plasma pressure (the terms $c_s^2\Delta n$ and $c_s^2(\partial_n^2/\partial z^2)$) in (4) and (5) can be neglected

$$\psi = \frac{1}{t_0 - t}\,\phi\!\left(\frac{r}{(t_0 - t)^{2/3}}\right); \qquad \delta n = \frac{1}{(t_0 - t)^{4/3}}\,V\!\left(\frac{r}{(t_0 - t)^{4/3}}\right). \quad (6)$$

The characteristic reciprocal time of development of the supersonic collapse is

$$\gamma \sim \omega_p\left(\frac{W}{nT}\,\frac{m}{M}\right)^{1/2}\frac{v_{Te}}{c} . \quad (7)$$

If the collapse develops from the subsonic states, then the characteristic initial dimension of the cavern is

$$L_0 = \frac{c}{\omega_p}\left(\frac{Mc^2}{T}\right)^{1/2} .$$

and the energy contained in it is

$$\mathcal{E}_0 \approx \left(\frac{c}{\omega_p}\right)^3\left(\frac{Mc^2}{T}\right)^{1/2} nT.$$

If the collapse is supersonic from the very beginning ($W_0/nT \gg T/Mc^2$), then the initial dimensions of the caverns can vary in the range

$$\frac{c}{\omega_p}\left(\frac{nT}{W_0}\right)^{1/2} < L_0 < \frac{c}{\omega_p}\left(\frac{c}{v_{Te}}\right)\left(\frac{W}{nT}\,\frac{m}{M}\right)$$

The cavern contains an energy $\mathcal{E}_0 \approx W_0 L_0^3$. The collapse introduces into the plasma wave a damping γ. The cavern develops as an entity until the oscillation intensity W in it reaches a value $nT(\omega_{He}/\omega_p)$, after which the high-frequency field inside the cavern turns out to be unstable to automodulation excitation of Langmuir waves with $kr_D \sim (\omega_{He}/\omega_p)^{1/2}$. As a result of the development of this instability, the initial cavern breaks up into a set of "ordinary" Langmuir randomly-oriented caverns of the dipole type.[2-4] The magnetic field exerts no significant influence on the collapse of these secondary caverns. The total number of secondary caverns produced in the decay of the primary cavern is, in a typical case, of the order of $(c/v_{Te})^3$.

Following the decay of the primary cavern, comparable fractions of the energy are contained in the secondary caverns and are returned to the weak-turbulent kinetic regime in the form of waves with wave numbers $k \sim (1/r_D)(\omega_{He}/\omega_p)^{1/2}$. If $\omega_{He}/\omega_p > m/M$, then the entire energy that falls in the secondary caverns is dissipated, and in the opposite case the fraction of the dissipated energy is smaller by a factor of $\omega_{He}M/\omega_p m$. Owing to its large dimensions, the primary cavern cannot lose energy in the form of Landau damping; the entire damping takes place in the secondary caverns. Since the dimension of the secondary cavern is bounded from above by its initial dimension $r_D\sqrt{\omega_p/\omega_{He}}$, only electrons having an energy smaller than $T(\omega_p/\omega_{He})$ can participate in the damping of the secondary caverns. This leads to the important conclusion that the magnetic field limits the growth of the "tails" of the electron distribution function to the value $T(\omega_p/\omega_{He})$, and at $\omega_{He} \sim \omega_p$ the collapse will not lead to the formation of fast electrons.

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