Concerning the possibility of verifying $SU(3)$-symmetry breaking in the reactions $e^+e^- \to 3\pi$ and $e^+e^- \to \pi^0\gamma$

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(Submitted March 3, 1975)
ZhETF Pis. Red. **21**, No. 8, 497–499 (April 20, 1975)

It is shown that a minimum in the cross sections of the reactions $e^+e^- \to 3\pi$ and $e^+e^- \to {}^0\gamma$ is predicted by the "mass mixing" model past the ϕ resonance, and by the "current mixing" model ahead of the ϕ resonance.

PACS numbers: 13.65., 11.30.Q

1. The interference of the ω and ϕ contributions to the reaction $e^+e^- \to 3\pi$ near the ϕ resonance will be experimentally studied in the nearest future.[1]

What can the sign of this interference tell us? Insofar as we know, this question has not been considered in the literature. We therefore wish to call attention in this article to the fact that the sign of the $\omega-\phi$ interference, which determines the position of the minimum in the cross section of the reaction $e^+e^- \to 3\pi$, is an effective means of choosing among the $SU(3)$-symmetry breaking models.

We shall show that

a) the "mass mixing" model[2–4] predicts a minimum past the ϕ resonance, and

b) the "current mixing" model[3,4] predicts a minimum ahead of the ϕ resonance.

These conclusions are true if the ϕ meson is connected with particles that do not contain strange quarks, mainly as a result of a slight deviation of the $\omega-\phi$ mixing angle from the "ideal" (quark) value.[5]

2. The cross section of the reaction $e^+e^- \to 3\pi$, with allowance for the contributions of the ω and ϕ resonances, is given by

$$\sigma(s) \sim \left| \frac{g_{\omega\rho\pi}}{f_\omega} \frac{m_\omega^2}{s - m_\omega^2} + \frac{g_{\phi\rho\pi}}{f_\phi} \frac{m_\phi^2}{s - m_\phi^2} \right|^2 \tag{1}$$

f_V are the constants of the $\gamma \leftrightarrow V$ transitions ($V = \omega, \phi, \rho$). For the $\phi \to 3\pi$ and $\omega \to 3\pi$ vertices we use without loss of generality the model of Gell-Mann, Sharp, and Wagner.[6] It is seen from (1) that the position of the minimum of $\sigma(s)$ relative to the ϕ resonance is determined by the sign of the ratio of the constants[7]

$$R = f_\omega g_{\phi\rho\pi} \big/ f_\phi g_{\omega\rho\pi} \; . \tag{2}$$

For the $VV\pi$ interaction, we assume that the "nonet symmetry" is valid.[2,5] This assumption holds true, for example, in the "naive" quark model[8] and makes it possible to relate the constants $g_{\phi\rho\pi}$ and $g_{\omega\rho\pi}$.[5,8]

In the mass-mixing model we have

$$g_{\phi\rho\pi} = g_{\omega\rho\pi} \operatorname{tg}(\theta - \theta_q). \tag{3}$$

θ_q is the ideal $\omega-\phi$ mixing angle; $\sin^2\theta_q = 1/3$, $|\theta_q| = 35.3°$. The angle θ is determined from the phenomenological mass formula of Gell-Mann and Okubo, which can be represented in the form

$$\sin^2\theta_q = \frac{1}{3}\left(1 - \frac{m_\omega^2 - m_\rho^2}{m_\phi^2 - m_\omega^2} + 2\frac{m_\phi^2 + m_\omega^2 - 2m_{k^*}^2}{m_\phi^2 - m_\omega^2}\right). \tag{4}$$

The experimental mass values yield $|\theta| \approx 40°$. The constants f_ω and f_ϕ are also connected via the $\omega-\phi$ mixing angle. In this case[4] $f_\omega/f_\phi = -\cot\theta$. Thus, relation (2) takes the form

$$R = -\operatorname{ctg}\theta \operatorname{tg}(\theta - \theta_q), \tag{5}$$

$$|\theta| - |\theta_q| \approx 5°, \tag{6}$$

which corresponds to the value $R \approx -0.1$ and to a minimum of $\sigma(s)$ past the ϕ resonance at $\sqrt{s} - m_\phi \approx 40$ MeV.

The sign of the angle θ cannot be determined from the phenomenological mass formulas, see (4). But formula (5) does not depend on the leeway in its choice.

In the current-mixing model we have

$$g_{\phi\rho\pi} = \frac{m_\phi}{m_\omega} g_{\omega\rho\pi} \operatorname{tg}(\theta - \theta_q), \quad f_\omega/f_\phi = -\frac{m_\omega}{m_\phi} \operatorname{ctg}\theta. \tag{7}$$

Therefore formula (5) remains valid as before. The angle θ, however, must now be determined from (4) by replacing all of the m_i^2 by m_i^{-2}. Then $|\theta|$ turns out to be $\approx 29.5°$, and

$$|\theta| - |\theta_q| \approx -6°, \tag{8}$$

which leads to a minimum of $\sigma(s)$ ahead of the ϕ resonance at $\sqrt{s} - m_\phi \approx -50$ MeV and $R \approx 0.18$.

We emphasize that the signs of the differences (6) and (8), from which our conclusions follow, are well defined. This can be verified from mass formulas (4) and (7) by making the substitutions $m_i^2 \to m_i^{-2}$.

All that has been said pertains obviously to the reaction $e^+e^- \to \pi^0\gamma$. The only difference from the reaction $e^+e^- \to 3\pi$ is that here, besides the contributions of the ω and ϕ mesons, there is also a contribution of the ρ meson. Its value, within the framework of the vector-dominance model, is equal to the ω contribution. Therefore the minimum in the reaction $e^+e^- \to \pi^0\gamma$ should be expected to be approximately twice as close to the ϕ resonance than in the reaction $e^+e^- \to 3\pi$.

Certain details, a discussion of the predictions of other possible variants of $SU(3)$ breaking,[4,9] and a comparison of the value of $|g_{\phi\rho\pi}/g_{\omega\rho\pi}| = |g_{\phi\pi\gamma}/g_{\omega\pi\gamma}|$ with the data on the ϕ and ω decays into 3π and $\pi^0\gamma$ are reported in[10].

One of us (G.N.Sh.) is sincerely grateful to V.E. Balakin, L.M. Barkov, V.P. Smakhtin, E.P. Solodov, and S.I. Éidel'man for discussions.

[1] A. Quenzer, Proceedings of the Eight Recontre de Moriond on El. and Weak Int., March 4—16, 1974, France, p. 125; Report presented at the Session USSR Academy of Sciences, September, 1972.

[2] S. Okubo, Phys. Lett. 5, 165 (1963); S. L. Glashow, Phys. Rev. Lett. 11, 48 (1963); J. J. Sakurai, Phys. Rev. 132, 434 (1963).

[3] S. Coleman and H. J. Schnitzer, Phys. Rev. 134B, 863 (1964).

[4] N. M. Kroll, T.D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[5] S. L. Glashow and R. H. Sokolow, Phys. Rev. Lett. 15, 329 (1965).

[6] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Lett. 8, 261 (1962).

[7] F. M. Renard, Nucl. Phys. B82, 1 (1974).

[8] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Lett. 17, 412 (1966); M. Gourdin, Proceedings of the Eleventh Session of the Scottish Univ. Summer School in Physics, 1970, on Hadronic Int. of El. and Ph., p. 395.

[9] I. Kimel, Phys. Rev. D3, 2048 (1971).

[10] N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, Inst. of Mathematics preprint TP-81 (1974).

Erratum: Concerning the possibility of verifying $SU(3)$-symmetry breaking in the reactions $e^+e^-\to 3\pi$ and $e^+e^-\to\pi^0\gamma$

[JETP Lett. 21, No. 8, 229–230 (20 April 1975)]

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Pis'ma Zh. Eksp. Teor. Fiz. 22, No. 7, 404 (5 October 1975)

PACS numbers: 01.85.+s, 13.65.+i, 11.30.Qc

"$\mathrm{Sin}^2\theta_q$" in formula (4) should be replaced by "$\sin^2\theta$."