Asymptotic neutrino cross sections

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(Submitted March 12, 1975)
ZhETF Pis. Red. **21**, No. 8, 499–503 (April 20, 1975)

The asymptotic behavior of neutrino cross sections is discussed within the framework of field theory. It is shown that if strong interactions are described by a theory that has either asymptotic freedom or a fixed point, and if the weak interactions (in the four-fermion form) are taken into account only in first order, then the neutrino cross sections increase linearly with increasing neutrino energy.

PACS numbers: 13.15., 11.10.J

It is well known that if there is scaling in deep inelastic processes, then the cross sections σ_ν of the neutrino reactions

$$\nu(\bar{\nu}) + N \to \mu^-(\mu^+) + \text{hadrons} \tag{1}$$

increase linearly with increasing neutrino energy. However, if strong interactions are described within the framework of the field theory, then exact scaling is realized neither in theories with fixed points nor in the asymptotically free theories that are being extensively investigated of late. It is therefore of interest to study the asymptotic form of the cross section of the reaction (1) within the framework of field theory.

The behavior of σ_ν was investigated in[1] in asymptotically free theories and it was shown that

$$A(\ln s/m_N^2)^{-P} < \sigma_\nu/s < \frac{1}{\delta}B(\ln s/m_N^2)^{\delta}\ ;\ s \equiv 2m_N E_\nu \to \infty, \tag{2}$$

where A and B are certain constants, δ is an arbitrarily small positive parameter, and P is a constant of the order of unity, which can be calculated and depends on the structure of the scheme used to describe the strong interaction. Relation (2) was obtained in the four-fermion model of weak interactions with allowance for only first order in the weak interaction.

We shall show in the same approximation that the neutrino cross section behaves like

$$\sigma_\nu \sim C_s\ ;\quad s \to \infty \tag{3}$$

both within the framework of asymptotically free theories, where the deviations from scaling are logarithmic, and within the framework of theories with fixed point, where the deviations from scaling have a power-law behavior. Relation (3) is the consequence of the fact that the zeroth moment of the structure function $F_2(x, Q^2)$ is

$$F_2^{(0)}(Q^2) \equiv \int_0^1 dx\, F_2(x, Q^2) \to \text{const} \quad \text{as} \quad Q^2 \to \infty, \tag{4}$$

and that the asymptotic form of $\sigma_\nu(s)$ is connected in a definite manner with the asymptotic form of $F_2^{(0)}(Q^2)$.[1)] In addition to (4), it is assumed in the derivation of (3) that in the asymptotic region the cross section $\sigma_\nu(s)$ is a "smooth" (i.e., a nonoscillating) function of s.

We note that the constant C in (3) is of the order of $(G_F/2\pi)F_2^{(0)}\ (Q^2 \to \infty)$.

Thus, the asymptotic form of the neutrino cross section in the four-fermion model of the weak interaction is insensitive to the strong-interaction scheme, since it is determined in fact only by the integral (4). The opposite situation holds in the model with intermediate W boson: At $E_\nu \gg m_w$ the asymptotic form of σ_ν depends essentially on the behavior of the structure functions $F(x, Q^2)$ in the region of small x, and is different for different strong-interaction schemes. On the basis of relation (4) we can only state that at $E_\nu \gg m_w$ the neutrino cross section increases no faster than s and no slower than a constant.

Why is the question of the behavior of the cross section of the reaction (1) not solved in general "automatically"? In the case when only the ultraviolet behavior of the theory is determined, the quantities that can be calculated theoretically are not the structure functions themselves, but their moments $F^{(n)}(Q^2) \equiv \int_0^1 dx x^n F(x, Q^2)$ at large Q^2:

$$F^{(n)}(Q^2) \to \begin{cases} C(n)[\ln Q^2/m_N^2]^{-a_n} & \text{in asymptotically free theories} \quad (5a) \\ \tilde{C}(n)[Q^2/m_N^2]^{-\tilde{a}_n} & \text{in theories with fixed point} \quad (5b) \end{cases}$$

The neutrino cross sections are not expressed directly in terms of the moments.

2. We proceed now to prove relation (3). Neglecting terms of order m_N/E_ν, the cross section of reactions (1) is expressed in terms of the structure functions $F_i(x, Q^2)$ in the following manner

$$\sigma_{\nu,\bar{\nu}}(s) = \frac{G^2 s}{2\pi} \int_0 dQ^2 \int_{Q^2/s}^1 \frac{dx}{x} \left\{ \left(1 - y + \frac{y^2}{2}\right) F_2 - \frac{y^2}{2} F_L \mp y\left(1 - \frac{y}{2}\right) x F_3 \right\} \quad (6)$$

where $y = Q^2/sx$ and $F_L = F_2 - 2xF_1$.

Using the inequalities $F_2 \geq 2xF_1 \geq x|F_3|$, we easily obtain

$$\frac{G_F^2}{2\pi} f(s) > \sigma_{\nu,\bar{\nu}}(s) > \frac{1}{8} \frac{G_F^2}{2\pi} f\left(\frac{s}{2}\right), \quad (7)$$

where

$$f(s) = \int_0^s dQ^2 \int_{Q^2/s}^1 \frac{dx}{x} F_2(x, Q^2) \equiv \int_0^1 \frac{dx}{x} \int_0^{sx} dQ^2 F_2(x, Q^2). \quad (8)$$

We note that $df/ds = \int_0^1 dx F_2(x, Q^2 = sx)$ and the asymptotic form of the neutrino cross section (7) is thus determined by the zeroth moments of the structure function F_2, but not at fixed Q^2 as in (4), but at fixed ν equal to $s/2m_N$. This difficulty can be eliminated. To estimate df/ds, we calculate below the integral $\int_{m_N^2}^{M^2} (df/ds)(ds/s)$, and show that it approaches $\ln(M^2/m_N^2)$ at large M^2:

$$\int_{m_N^2}^{M^2} \frac{df}{ds} \frac{ds}{s} = \int_0^{m_N^2} \frac{dQ^2}{Q^2} \int_{Q^2/M^2}^{Q^2/m_N^2} dx F_2(x, Q^2) + \int_{m_N^2}^{M^2} \frac{dQ^2}{Q^2} \int_{Q^2/M^2}^1 dx F_2(x, Q^2) \quad (9)$$

It is easily seen that at large values of M^2 the first integral in the right-hand side of (9) is negligibly small in comparison with the second. Indeed,

$$\int_0^{m_N^2} \frac{dQ^2}{Q^2} \int_{Q^2/M^2}^{Q^2/m_N^2} dx F_2(x, Q^2) < \int_0^{m_N^2} \frac{dQ^2}{Q^2} \int_0^{Q^2/m_N^2} dx F_2(x, Q^2) = \text{const},$$

whereas

$$\int_{m_N^2}^{M^2} \frac{dQ^2}{Q^2} \int_{Q^2/M^2}^1 dx F_2(x, Q^2) \sim \ln M^2$$

(see the restrictions (11)):

$$\int_{m_N^2}^{M^2} \frac{dQ^2}{Q^2} \int_0^1 dx F_2(x, Q^2) > \int_{m_N^2}^{M^2} \frac{dQ^2}{Q^2} \int_{Q^2/M^2}^1 dx F_2(x, Q^2) > \int_{m_N^2}^{M^b m_N^{2-b}} \frac{dQ^2}{Q^2} \int_0^1 dx$$

$$\times \left[1 - \left(\frac{Q^2}{M^2 x}\right)^a\right] F_2(x, Q^2) = \int_{m_N^2}^{M^b m_N^{2-b}} \frac{dQ^2}{Q^2} \left[F_2^{(0)}(Q^2) - F_2^{(-a)}(Q^2)\left(\frac{Q^2}{M^2}\right)^a\right], \quad (10)$$

where a and b are positive parameters. At large Q^2, the moment $F^{(-\alpha)}(Q^2)$ is given by formula (5). If the parameter b in (10) is chosen smaller than $2\alpha/(\alpha + |\tilde{\alpha}(-\alpha)|)$, then, taking (5) into account, we see that the term $F^{(-\alpha)}(Q^2)[Q^2/M^2]^\alpha$ in the integral of (10) can be neglected in comparison with $F^{(0)}(Q^2)$, and consequently

$$C \ln \frac{M^2}{m_N^2} > \int_{m_N^2}^{M^2} \frac{dQ^2}{Q^2} \int_{Q^2/M^2}^1 dx F(x, Q^2) > \frac{bC}{2} \ln \frac{M^2}{m_N^2}, \quad (11)$$

where $C = F^{(0)}(Q^2 \to \infty)$. Comparing (1)—(11), we find that at large M^2

$$C \ln \frac{M^2}{m_N^2} > \int_{m_N^2}^{M^2} \frac{df}{ds} \frac{ds}{s} > \frac{bC}{2} \ln \frac{M^2}{m_N^2}. \quad (12)$$

The system of restrictions (12) is rigorous. In the derivation of (12) we used (4) and the assumption that the function $F^{(n)}(Q^2)$, which in general can be calculated at $\mathrm{Re}\, n \geq 0$, can be continued analytically into the region of small negative $\mathrm{Re}\, n$ up to $\mathrm{Re}\, n = -|n_0|$. The positive parameter α in (10) can be chosen arbitrarily provided that its value is smaller than $|n_0|$.

We now assume also that at large s, "when the asymptotic form has already been reached," the function df/ds is monotonic. It is difficult to justify this assumption rigorously, but it seems natural, since the presence of oscillations in the asymptotic region calls for special physical causes. Under this additional assumption, it follows immediately from (12) that $df/ds \to$ const, and by using (7) we arrive at the linear asymptotic form of the neutrino cross sections (3).

The author is grateful to B.L. Ioffe for discussions.

[1]A. Zee, F. Wilczek, and S. B. Treiman, Phys. Rev. D10, 2881 (1974).